Chapter 4 – Radiative Transfer
Radiative Transfer

• Primary method of energy exchange between the Earth and rest of universe.

• Transfers occur between the atmosphere and Earth, and between layers of the atmosphere.
The Spectrum of Radiation

- Electromagnetic Radiation
  - Travels at speed of light ($3 \times 10^8$ m/s).
  - Consists of a variety of frequencies and wavelengths.
Quantum Theory

- Electromagnetic radiation is made up of photons, or packets of energy.

- Photon energy = \( W = hf = \frac{hc}{\lambda} \)
  - \( \lambda = \) wavelength
  - \( f = \) frequency
  - \( c = \) speed of light.

- Energy is inversely proportional to wavelength.

- Energy is directly proportional to frequency.
Nomenclature

• Radiant flux: Rate of energy transfer by electromagnetic radiation.
  • Units: Energy/time = J/s = W = Watts.
  • Example: Radiant flux from sun = $3.9 \times 10^{26}$ W.

• Irradiance: Radiant flux/Area = \(E\)
  • Units: W/m\(^2\)
  • Example: Irradiance at outermost disk of sun.

• Monochromatic irradiance: \(E = E / \lambda\)
  • Units: W/m\(^2\) m = W/m\(^2\) um.
Diffuse and Direct Radiation

• Diffuse Radiation: Radiation emanating from a source that subtends a finite arc of solid angle.
  • Scattered radiation is an example.

• Parallel Beam Radiation: Emission from a concentrated source.
  • Radiance approaches infinity and the angle subtended by the source approaches zero.
  • Direct beam radiation
Measurement of Radiation

• Black and white surface

• Black absorbs radiation, white reflects radiation

• Amount of radiation received and absorbed determines the differences in the rate of increase of temperatures between the two surfaces.

• Pyranometer
Blackbody Radiation

• Hypothetical body comprising a sufficient number of molecules absorbing and emitting electromagnetic radiation in all parts of the spectrum so that:
  • All incident radiation is completely absorbed.
  • Maximum possible emission is realized in all wavelength bands, in all directions (isotropic).

• Planck's law: Amount of radiation emitted by a blackbody.
  • Uniquely determined by its temperature.
Wien’s Displacement Law

• $\lambda_{\text{max}} = 2880 \text{ um K/T}$

• Wavelength of peak emission for a blackbody at temperature T.

• Estimate the temperature of a radiation source from its emission spectrum.
  • If we assume a blackbody source, then knowing the emission spectrum, we can deduce T.
More Blackbody Spectra

Fig. 6.4  Normalized blackbody spectra representative of the sun (left) and earth (right), plotted on a logarithmic wavelength scale. The ordinate is multiplied by wavelength in order to make area under the curves proportional to irradiance. [Adapted from R. M. Goody, "Atmospheric Radiation," Oxford Univ. Press (1964), p. 4.]
Blackbody Spectrum cont.

- Peak for sun is in blue, but asymmetry of spectrum gives more radiation toward yellow side.
- Earth emits @ ~255 K
- Sun concentrated in visible and near infrared, planets and their atmospheres largely confined to infrared.
- Note: Curves barely overlap
  - Treat solar (shortwave) radiation separately from terrestrial (longwave) radiation.
Calculations

• Irradiance at the top of the Earth’s atmosphere

• Equivalent Blackbody Temperature of Sun

• Equivalent Blackbody Temperature of Earth

  • This calculation assumes the Earth does not have an atmosphere and references the image on the next slide
Absorptivity and Emissivity

• Blackbody radiation is an upper limit to the amount of radiation a real substance may emit at a given temperature.
  • Real world radiation < Blackbody

• At any given wavelength, \( \lambda \), we can define the Emissivity, \( \varepsilon \equiv \frac{E_\lambda}{E^{*}_\lambda} \)
  • Emissivity is a measure of how strongly a body radiates at that wavelength.
  • \( \varepsilon_{\text{blackbody}} \equiv 1 \) at all wavelengths.
  • \( 0 < \varepsilon_{\text{real substance}} < 1 \)

• “Grey body” emissivity: \( \varepsilon \equiv \frac{E}{E^{*}} = \frac{E}{\sigma T^4} \) and \( E_{\text{grey}} = \varepsilon \sigma T^4 \)
  • “Grey” comes from the neglect \( \lambda \) of wavelength dependence of the emissivity.
  • Most real substances behave as grey bodies and have an emissivity that is different from 1.

• Absorptivity, \( a_\lambda \equiv \frac{\text{irradiance absorbed}}{\text{irradiance incident}} \)
  • “grey body” absorptivity = \( a \)
  • \( a_{\text{blackbody}} = 1 \)
Kirchhoff's law

• Kirchhoff's law: Materials that are strong absorbers at a particular $\lambda$ are also strong emitters at that $\lambda$.

• $a = \varepsilon$

• $a_\lambda = \varepsilon_\lambda$

• Weak absorbers = weak emitters

• Applies to gases like our atmosphere.
Reflectivity and transmissivity

- What happens to the part not absorbed? It is reflected.
- \[ E_\lambda \text{ (incident)} = E_\lambda \text{ (absorbed)} + E_\lambda \text{ (reflected)} \]
- Dividing by \( E_\lambda \text{ (incident)} \) yields:
  - \[ \frac{E_\lambda \text{ (incident)}}{E_\lambda \text{ (incident)}} = \frac{E_\lambda \text{ (absorbed)}}{E_\lambda \text{ (incident)}} + \frac{E_\lambda \text{ (reflected)}}{E_\lambda \text{ (incident)}} \]
  - \[ 1 = \frac{E_\lambda \text{ (absorbed)}}{E_\lambda \text{ (incident)}} + \frac{E_\lambda \text{ (reflected)}}{E_\lambda \text{ (incident)}} \]
  - \[ 1 = a_\lambda + r_\lambda \]
  - Reflectivity, \( r_\lambda = \frac{E_\lambda \text{ (reflected)}}{E_\lambda \text{ (incident)}} \)
    - Large \( r_\lambda \) = small \( a_\lambda \) and vice versa.
- More generally, for non-opaque media, some of the incident radiation is transmitted.
  - Transmissivity, \( \tau_\lambda = \frac{E_\lambda \text{ (transmitted)}}{E_\lambda \text{ (incident)}} \)
  - \( a_\lambda + r_\lambda + \tau_\lambda = 1 \)
Greenhouse Effect

- Solar radiation essentially passes through to surface.
- Atmosphere absorbs some of IR emitted by the surface and emits it back.
- Surface must warm up even faster to emit enough radiation so that output can match the input
  - Radiative equilibrium
Atmospheric Absorption of Solar Radiation

• Absorption of parallel beam radiation is proportional to the number of molecules of gas along the path.
  • For now, we are going to ignore scattering of photons out of the beam.

• This can be expressed as: \( da_\lambda = -dE_\lambda / E_\lambda = -K_\lambda \rho \sec \varphi \, dz \)
  • where:
    • \( da_\lambda \) is the absorption that occurs through the layer.
    • \( \rho \) is the density of the gas
    • \( \sec \varphi \, dz \) is the path length (see online lecture)
      • \( \rho \sec \varphi \, dz \) is the mass per unit area for a small \( dz \) (think about units)
    • \( K_\lambda \) is the absorption coefficient of the gas [m\(^2\)/kg]
      • How efficient the gas is as an absorber
      • Also called the absorption cross-section
      • \( K_\lambda \) is a function of the temperature of the gas, pressure, and composition of the gas.

• There are three ways to change the amount of absorption:
  • Change the density of the gas (more absorbers per unit area)
  • Change the path length
  • Change the absorption coefficient
Beer’s Law and Transmissivity

- See in class lecture deriving Beer’s law and how it relates to the transmissivity

- Beer’s law is an equation for the cumulative absorption or how much of the radiation remains after passing through a given thickness of the atmosphere.
\[ \text{Field} \]

[From J. Appl. Meteor., 12, 376, (1973).]

\[ \lambda = 0.58 \, \mu m \]

\[ \lambda = 0.40 \, \mu m \]
Atmospheric Scattering of Solar Radiation

- $d_{s \lambda} = $ Fraction of parallel beam radiation that is scattered when passing downward through a layer of infinitesimal thickness.

- This can be expressed as: $d_{s \lambda} = -dE_{\lambda} / E_{\lambda} = -K_{\lambda} N \sigma \sec \varphi \, dz$
  
  where:
  
  - $d_{s \lambda}$ is the scattering that occurs through the layer.
  - $N$ is the number of particles per unit volume of air (particle density).
  - $\sec \varphi \, dz$ is the path length (see online lecture).
  - $\sigma$ scattering cross-sectional area of each particle.
  - $K_{\lambda}$ is the scattering coefficient of the gas [m$^2$/kg]
    - How efficient the gas is at scattering
    - $K_{\lambda}$ is a function of the size parameter and the refractive index of the particles in the gas.

- There are four ways to change the amount of scattering:
  
  - Change the number of particles per unit volume.
  - Change the path length.
  - Change the scattering coefficient.
  - Change the scattering cross-section of the particles.

- By following the approach we took with absorption, you can come up with an equation similar to Beer’s law, but for scattering.
Scattering, $x$, and $\lambda$
For size parameters much less than 1, $K_\lambda \propto x^4$ and $K_\lambda \propto \lambda^{-4}$

- Rayleigh scattering of solar (shortwave) radiation.
- Scattered radiation is evenly divided between forward and back-scattered hemispheres.
- $K_\lambda (\text{blue, } \lambda = 0.47 \text{ um})/K_\lambda (\text{red, } \lambda = 0.64 \text{ um}) = (0.64/0.47)^4$
- Short $\lambda$ light is scattered more than long $\lambda$.
- Short $\lambda$ is preferentially scattered.
  - Responsible for blue sky.
- Longer $\lambda$ is more readily transmitted
  - Reddish or orange appearance of objects.
  - Especially around sunrise and sunset.
  - Path length through the atmosphere is long.
Scattering, $\chi$, and $\lambda$
Microwave Scattering

• Microwave scattering by raindrops also falls in the Rayleigh regime.

• For a given $\lambda$, $K_\lambda \propto x^4$.

• Sharp increase in $K_\lambda$ with increasing drop size.

• Makes it possible to discriminate between precipitation and cloud drops (radar).

• Why not use infrared radiation?
Doppler Radar Bands

- **S band**
  - 8-15 cm wavelength
  - Not easily attenuated
  - NWS radars – 10 cm
- **C band**
  - 4-8 cm wavelength
  - More easily attenuated
  - Smaller dish sizes make them more affordable
  - TV stations
- **X band**
  - 2.5-4 cm wavelength
  - Easily attenuated, useful for short range observation
  - Cloud development
  - DOWs
- **K band**
  - 0.75-1.2 cm wavelength
  - Similar to X band, but even more sensitive
  - Shares space with police radars.
Scattering, $x$, and $\lambda$
\[ x > 50, \ K \lambda \approx 2 \]

- Angular distribution of scattered radiation described by principles of geometric optics.

- Scattering of visible radiation by cloud droplets, rain drops, ice particles.

- Rainbows, halos, etc.
0.1 < x < 50

- $K_{\lambda}$ exhibits oscillatory behavior
- Angular distribution of radiation very complicated and varies rapidly with the size parameter.
- Forward scattering predominates over back scattering.
- Scattering of sunlight by smoke, smog, dust.
Scattering, $x$, and $\lambda$

![Graph showing the relationship between $r$ (in $\mu m$) and $\lambda$ (in $\mu m$) with various scattering processes and particle sizes.](image)
Extinction

- Equations for scattering and absorption are very similar.

- In fact, they can be made to be identical with the following equation:

  \[ K_\lambda (\text{Extinction}) = K_\lambda (\text{Scattering}) + K_\lambda (\text{Absorption}) \]

- This equation gives the combined effect of scattering and absorption in depleting the intensity of radiation passing through the layer.