1. Isothermal Sound Waves

The first time someone tried to derive the behavior of sound waves, it was assumed that they behave isothermally, rather than adiabatically. Thus, we would use dT/dt = 0, rather than  $d\theta/dt = 0$ . Making this change, we would arrive at

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0$$
(1a,b)
$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + p \frac{\partial u}{\partial x} = 0$$

(a) Show that an acceptable basic state is  $u = \overline{u}$ ,  $p = \overline{p}$ ,  $\rho = \overline{\rho}$ , where all three fields are constants.

(b) Assume the isothermal sound waves are given by perturbations about this basic state:  $u = \overline{u} + u', p = \overline{p} + p', \rho = \overline{\rho} + \rho'$ . Substitute these forms for u, p, and  $\rho$  into (1a,b) and obtain the corresponding two linearized equations for the perturbation fields.

(c) If we eliminate variables in the two linearized equations to get a single equation in p', we then have

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right) \left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right) p' - \frac{\overline{p}}{\overline{\rho}}\frac{\partial^2 p'}{\partial x^2} = 0$$
(2)

As mentioned in class, the form  $\left(\frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial x}\right)$  is a *differential operator*. The derivatives in it operate on whatever is to the right of it. When we see the form twice in a row, as in (2), it means we apply the rightmost  $\left(\frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial x}\right)$  to p', and then do it again with the next  $\left(\frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial x}\right)$ . This is analogous to considering  $\frac{\partial^2 p'}{\partial x^2}$  as  $\left(\frac{\partial}{\partial x}\right) \left(\frac{\partial p'}{\partial x}\right)$ , where first we compute  $\left(\frac{\partial p'}{\partial x}\right)$ , and then take  $\left(\frac{\partial}{\partial x}\right)$  of that result.

With that in mind, assume a wave solution,  $p' = A \exp\{ik(x - ct)\}\)$ , and derive the relationship for phase speed c in terms of  $\overline{u}$ ,  $\overline{p}$ , and  $\overline{p}$ .