1. Isothermal Sound Waves - Part 1

The first time someone tried to derive the behavior of sound waves, it was assumed that they behave isothermally, rather than adiabatically. Thus, we would use dT/dt = 0, rather than dθ/dt = 0. Making this change, we would arrive at

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\frac{\partial p}{\partial x}=0$$
$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+p\frac{\partial u}{\partial x}=0 \qquad \text{(1a,b)}$$

(a) Show that an acceptable basic state is $u=\overline{u}$, $p=\overline{p}$, $\rho=\overline{\rho}$, where all three fields are constants.

(b) Assume the isothermal sound waves are given by perturbations about this basic state: $u=\overline{u}+u'$, $p=\overline{p}+p'$, $\rho=\overline{\rho}+\rho'$. Substitute these forms for u, p, and ρ into (1a,b) and obtain the corresponding two linearized equations for the perturbation fields.

(c) If we eliminate variables in the two linearized equations to get a single equation in p', we then have

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)p'-\frac{\overline{p}}{\overline{\rho}}\frac{\partial^2 p'}{\partial x^2}=0 \qquad (2)$$

As mentioned in class, the form $\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)$ is a *differential operator*. The derivatives in it operate on whatever is to the right of it. When we see the form twice in a row, as in (2), it means we apply the rightmost $\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)$ to p', and then do it again with the next $\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)$. This is analogous to considering $\frac{\partial^2 p'}{\partial x^2}$ as $\left(\frac{\partial}{\partial x}\right)\left(\frac{\partial p'}{\partial x}\right)$, where first we compute $\left(\frac{\partial p'}{\partial x}\right)$, and then take $\left(\frac{\partial}{\partial x}\right)$ of that result.

With that in mind, assume a wave solution, $p'=A\exp\{ik(x-ct)\}$, and derive the relationship for phase speed c in terms of $\overline{u}$, $\overline{p}$, and $\overline{\rho}$.