

Group velocity for Rossby waves (simple case, $U=0$ and $l = 0$)

$$(\vec{c}_g)_x = \frac{\partial v}{\partial k}$$

What is v ? We have $v = ck = Uk - \beta k / (k^2 + l^2)$, so

$$(\vec{c}_g)_x = \frac{\partial v}{\partial k} = \frac{\partial}{\partial k} \left\{ Uk - \frac{\beta k}{k^2 + l^2} \right\} = \frac{\partial}{\partial k} \left\{ -\frac{\beta k}{k^2 + l^2} \right\}$$

or,

$$(\vec{c}_g)_x = \left\{ -\frac{\beta}{k^2 + l^2} \right\} \frac{\partial}{\partial k} \{k\} - k \frac{\partial}{\partial k} \left\{ \frac{\beta}{k^2 + l^2} \right\}$$

$$(\vec{c}_g)_x = \left\{ -\frac{\beta}{k^2 + l^2} \right\} - \frac{\beta k}{(k^2 + l^2)^2} \frac{\partial}{\partial k} \{-k^2\}$$

$$(\vec{c}_g)_x = \left\{ -\frac{\beta}{k^2 + l^2} \right\} + \frac{2\beta k^2}{(k^2 + l^2)^2} = \frac{2\beta k^2}{(k^2 + l^2)^2} - \left\{ \frac{\beta(k^2 + l^2)}{(k^2 + l^2)^2} \right\}$$

$$(\vec{c}_g)_x = \left\{ \frac{\beta(k^2 - l^2)}{(k^2 + l^2)^2} \right\}$$

For $l = 0$, then

$$(\vec{c}_g)_x = \left\{ \frac{\beta}{k^2} \right\}$$

This is the opposite sign of

$$c_x = -\left\{ \frac{\beta}{k^2} \right\}$$

for the same case. Thus the wave form moves to the west, but the energy propagates to the east.