Analysis of the 500 mb height fields and waves: testing Rossby wave theory

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ABSTRACT

According to Rossby wave theory, smaller wave numbers are associated with increased wave speed and decreased amplitude. Analysis of the 500 millibar heights and flow in the Northern and Southern Hemispheres and the zonally averaged wind by latitude over the course of the time period from late August through early November 2008 will test this theory. Most of the results do not correlate well. It is believed that errors in the methodology and analysis give the impression that Rossby wave theory has failed, when in reality, it may be more difficult to verify than what is done in this study.

I. Introduction

A study of the 500 mb height field and the waves present in the Northern and Southern Hemispheres can reveal a great deal about how waves behave in the atmosphere; so also can the zonally averaged winds around the globe. Rossby wave theory dictates that there is a relationship between wavelength and phase speed. This will be tested by observing the properties of waves in the 500 mb height field in both hemispheres. Analyses of the waves in the Northern and Southern Hemisphere will indeed show differences and similarities.

Section 2 addresses the source of data and the methodology of analysis. Analyses of the wave properties occur in Section 3. Section 3a addresses the speed at which the wave patterns move and the relationship between maximum zonally averaged wind speed and wave propagation speed. Section 3b addresses how wave number changes with time, which helps determine the length of any identifiable patterns, and the relationship between wave number and amplitude is also analyzed. Section 3c addresses the changes in average wave amplitude over time. Section 3d analyzes the evolution of maximum zonally averaged wind at two levels in the upper troposphere, and also relates those winds to the growth and decay of the waves. Conclusions follow in Section 4.

2. Methodology and Data Sources

The data used in this study came from the Iowa State University Weather Products web site (http://www.meteor.iastate.edu/wx/data/). Specifically, products used include 500 mb heights in both the Northern and Southern Hemispheres and the zonally averaged wind. Each product is calculated and produced using 0000 UTC data each day. Over a course of 82 days from 19 August 2008 through 8 November 2008, data was collected from each source. The specific data collected included, for each Hemisphere, the dominant integral wave number, the average amplitude of all waves, the average phase speed of the waves, and the maximum value of the wind speed at 500 mb and in the 300-150 mb layer. To make observations of wave number and amplitude objective, specific contours and latitude circles were used. The 50 degree latitude circle was used for both Hemispheres, and the 5580 m and 5280 m height contour was used for the Northern and Southern Hemisphere, respectively. The wave number was determined by the number of times the target contour in each Hemisphere crossed the 50 degree latitude circle. Then, the highest and lowest heights along the 50 degree latitude circle for each wave were averaged to determine the amplitude for each wave. The amplitude of each wave was averaged to determine the average amplitude for the waves for each day. Phase speed was determined by the change in longitude of the intersection of the target contour and the 50 degree latitude circle for each eligible wave each day. For a given day, a center-differencing scheme was used: the difference in longitude of the intersection between the day after and the day before were averaged to determine the phase speed for the given day.

3. Analysis

a. Upper air wind speed vs. wave speed

Attempts at finding a relationship between zonal wind and wave speed revealed that Rossby wave theory did not hold well for the data set. The average wave speed over the 82 days was 6.07 degrees/day (5.02 m/s) in the Northern Hemisphere and 12.51 degrees/day (10.34 m/s) in the Southern Hemisphere. For reference, these speeds indicate waves would travel around the 50 degree latitude circle in about 59 and 29 days in the Northern and Southern Hemispheres, respectively. Therefore, it can be said that, on average, waves traveled just over twice as fast in the Southern Hemisphere as in the Northern Hemisphere. Waves traveled from west to east in both Hemispheres.

To explain the difference in wave speed between the two Hemispheres, the average of the maximum wind speed at 500 mb was calculated for each Hemisphere. The average was 23.54 m/s and 33.32 m/s in the Northern and Southern Hemispheres, respectively. Although the average for the Southern Hemisphere was greater than that for the Northern Hemisphere, it was only so by a factor of ~1.42, which does not fully explain the difference in wave speed between the Hemispheres. For the maximum wind in the 300-150 mb layer, the average was 36.29 m/s and 52.16 m/s for the Northern and Southern Hemispheres, respectively, a difference of a factor of ~1.44. Rossby wave theory for a barotropic atmosphere reveals that wave speed is proportional to background wind speed, but also proportional to wavelength. Therefore, to account for the difference, if Rossby wave theory was correct in this sense, a contribution from wavelength would also be evident. This will be discussed later.

To determine a relationship between background wind speed and wave speed, the maximum wind at 500 mb was plotted against wave speed for each Hemisphere (figures 1 and 2). Using Microsoft Excel 2007 to plot each data set and to determine a line of best fit, it was determined that in neither Hemisphere did any significant relationship between 500 mb winds and wave speed exist, as $R^2 < 0.01$ for each Hemisphere (other types of regression, such as exponential and quadratic regression were attempted, but all gave correlation coefficients of $R^2 < 0.1$, indicating those regressions would give no better fit than a line). Also, note that the line of best fit had a negative slope, indicating a decrease in wave speed as 500 mb winds increased. However, the slopes were small in magnitude (less than -0.03 in each Hemisphere), and one must keep in mind that the very small correlation coefficient for the lines makes it very difficult to say with certainty that any trend exists. Note that the average wave speed was much less than the average wind speed at 500 mb in each Hemisphere. This does agree with Rossby wave theory since Rossby wave theory indicates that wave phase speed, c, is related to basic state wind speed by

$$c = \bar{u} - \beta/(k^2 + l^2),$$

where $\bar{u}$ is the background wind speed, $\beta = \partial f/\partial y$ is the change in absolute vorticity in the meridional direction, and k and l are the horizontal wave numbers given by $l = 2\pi/L_y$ and $k = 2\pi/L_x$ where L_y and L_x are the wave numbers in the meridional and zonal directions, respectively. Since $\beta > 0$, $c < \bar{u}$ always. This is evident in figures 1 and 2. However, Rossby wave theory also indicates that phase speed should increase with increasing

500 mb wind speed. As has been shown, this is not the case. Therefore, what little the plots for 500 mb winds against wave speed show, it contradicts Rossby wave theory, albeit weakly.

Figures 3 and 4 show a plot of maximum wind in the 300-150 mb layer against wave speed. Similar to the plots using 500 mb winds, there is a very weak correlation between 300-150 mb winds and wave speed: the line of best fit had $R^2 < 0.04$ for each Hemisphere. The slopes of the lines were both negative, but small in magnitude, with a value of -0.103 in the Southern Hemisphere and -0.012 in the Northern Hemisphere. Also similar to the plots using 500 mb winds, the average wave speed was much less than the average of the maximum wind in the 300-150 mb layer. To explain the weak correlation between 300-150 mb winds and wave speed, 500 mb wind speed was plotted against 300-150 mb wind speed, and a line of best fit was added (figures 5 and 6). The correlation coefficients for these regression lines were much larger, with R^2 values of 0.481 and 0.216 for the Northern and Southern Hemispheres, indicating a decent, although still somewhat weak, relationship between the winds at each layer. The relationship was positive in each Hemisphere, however, with slopes of +0.930 and +0.673 in the Northern and Southern Hemisphere, which shows that winds generally increased in each layer together. Therefore, since there was little correlation between 500 mb winds and wave speed and a moderate correlation between 500 mb and 300-150 mb winds, one would not necessarily expect there to be much relationship between 300-150 mb winds and wave speed either. Possible reasons for the low correlation coefficient values and non-one-to-one slopes between 500 mb and 300-150 mb winds include the fact that the maximum wind at each level did not necessarily occur at the same latitude, differences in density of the atmosphere at the two levels, the fact that there may not be any mixing that occurs between 500 mb and the 300-150 mb layer, and that if there is a relationship, it may be non-linear or chaotic.

Rossby wave theory says that these waves are dispersive, or that wave speed varies with wave number. The dependence of wave speed on zonal wind was removed, leaving wave speed alone. This is plotted against integral wave number in figures 7 and 8. The plots reveal that, in the Southern Hemisphere, wave speed increases with wave number, whereas the opposite is true in the Northern Hemisphere (although again the correlation is very weak, with R^2 values for lines of best fit to the data of 0.030 and 0.065 in the Northern and Southern Hemispheres). Since (non-integral) wave number is inversely proportional to wavelength and wave speed is inversely proportional to wave number, then since the non-integral and integral wave numbers are directly proportional (the authors determined integral wave number in this study), then Rossby wave theory states that wave speed is inversely proportional to wavelength (recall the formula for wave speed: $c = u - \beta/(k^2 + l^2)$, where $l = 2\pi/L_y$ and $k = 2\pi/L_x$). This means that waves of longer wavelength should travel at slower speeds than those of shorter wavelength. Lesser integral wave numbers imply longer wavelengths and vice versa, which indicates that wave speeds for smaller wave numbers should be less than wave speeds for larger wave numbers. The plot of wave speed against wave number agrees with Rossby wave theory in the Southern Hemisphere and disagrees with it in the Northern Hemisphere. Again, the relationship as determined by the correlation coefficient of the line of best fit for the plot in each Hemisphere was very weak, so it is difficult to say whether Rossby theory is definitely violated or not. What can be said by all of the plots and their respective lines of best fit is that in this study, no strong relationships were discovered between upper air winds and wave speed, and wave number and wave speed.

b. Changes in dominant wave number over time and amplitude vs. wave number

The length of time over which an identifiable wave pattern was analyzed. This was done by looking at the dominant integral wave number and how long a pattern lasted, or by evaluating the change in wave number over time. Figures 9 and 10 show that there were many periods in which a certain wave number dominated. Starting with the Northern Hemisphere, there were stretches where the wave number remained constant for as long as eight days (as it did at 3 from 9/27 through 10/4). The wave number was most variable throughout the beginning of the period of study and the end of the period. The variability of the wave number did not vary much from day to day, and there were few spans of days where the wave number varied by more than one from the previous day. It was interesting to observe the jump in wave number on the Autumnal Equinox in the

Northern Hemisphere. After the Equinox, there was a significant decrease in wave number. Afterward, the pattern resembled that of the first half of the period of study.

The Southern Hemisphere showed a similar pattern, but there was more variation from day to day. There were also more periods during which the wave number jumped by more than one in 24 hours. It was also noted that there was greater consistency in wave number towards the end of the period of study and more variability at the beginning. The longest streak of constant wave number in the Southern Hemisphere was six days, during which the wave number remained at four. One interesting pattern occurred a few days after the Autumnal Equinox, when the wave number became fairly consistent at around three or four in the Southern hemisphere, only changing to values outside the range from three to five twice.

Along with the changes in wave number over time, observations tended to be consistent with the synoptic time scale. According to Rossby wave theory, wave number depends on wavelength. The speed of wave propagation is the ratio of frequency to wave number. The average length, according to Rossby wave theory, should be less than a day, but it was observed in this study to be closer to a few days. Most patterns in both hemispheres were consistent with that. As can be expected, some lasted longer than others, and some were relatively short lived.

It was also attempted to find a relationship between integral wave number and average amplitude. The lines of best fit for the plots in each hemisphere, although having larger correlation coefficients than other plots in this study, are still very low, suggesting a meager linear relationship between average amplitude and wave number. Another observation of the plots in figures 11 and 12 showed that average amplitudes were noticeably larger in the Southern Hemisphere than in the Northern Hemisphere. Note also that the prominence of wave numbers three and four can be seen by the large number of points in the figure corresponding to those wave numbers. It was indeed rare to see a wave number of one or above five.

Rossby wave theory also indicates that shorter waves tend to have larger amplitudes. Shorter waves should also move faster than longer waves. This allows them to propagate faster and 'dig' themselves deeper within the flow allowing for higher amplitudes. As shown in figures 11 and 12, this was was not observed in the study.

c. Changes in average amplitude over time

In the Northern Hemisphere over the entire period of study, there was an overall trend towards increased average amplitude, as shown by the positively sloped line of best fit in figure 13. A few exceptions to this trend occurred for about one week in mid October when average amplitude decreased noticeably, and at the beginning of the analysis period in mid to late August, when there was no focused trend. Over the course of the later half of the period of study, the amplitude nearly doubled to about 220 m compared to the nearly static trend from late August to mid September, when the average amplitude hovered between 100 m and 150 m. It could be argued that there was a very gradual decrease during the first few weeks of September just before there began an upward climb in late September. The average amplitude from late August to mid through late September was about 112 m (based on values from 8/27 to 9/22, or 27 days), whereas the average increased strongly after the decrease in mid-October. An average of the dates from 10/28 to 11/8 was 206 m, an increase of 84% from the earlier season value of 112 m.

Sometimes it can be helpful to calculate running averages since doing so tends to reduce noise in the signal being investigated. Any significant trends occurring would then become rather prominent. The 3-day and 5-day running averages (figures 14 and 15) of these data points successfully reduce the noise of the graph, and reveal the overall gradual upward trend of the amplitude versus time as the Northern Hemisphere progressed into the colder regime of winter. There was also a more obvious oscillation, especially when looking at the 5-day running average. The relative damping of the oscillation in September evolved into growing oscillations in October; there also was a peak in August noticeable in both 3-day and 5-day running average as well.

The line of best fit showed an overall increase in amplitude of about 50 meters (slope of 0.6065 over the 82 days), with an R^2 value of 0.1231. The trend visually looks more polynomial in nature, and the growth in average amplitude as time progresses from September into October leads to a possible hypothesis of the affect of the Equinox damping the wavelike pattern in this plot. When a polynomial regression was applied, it was found that for increasing degrees of the polynomial, the correlation coefficient rose dramatically. For a sixth-degree polynomial, $R^2 = 0.456$, and the curve showed a much better fit than the line. This increase in R^2 with increasing degree of polynomial is to be expected to some extent since any periodic or wavelike function can be represented as a polynomial of infinite degree by using a Taylor Series. This supports the notion that the time series of average amplitude in the Northern Hemisphere is indeed wavelike. This also occurred in the Southern Hemisphere, the discussion of which follows.

In the Southern Hemisphere, on the other hand, there was a general decrease of amplitude throughout the period of study (figure 16). 3-day and 5-day running averages were also calculated for the Southern Hemisphere and are shown in figures 17 and 18. There was also considerably more variation or spread, with the trend being more subtle than in the Northern Hemisphere. The beginning of the period of study had lower values, with a period of growth through late August. The average of the first nine days was 142 m. Like the Northern Hemisphere, the average amplitude during the four week period of time between late August and mid to late September seemed to be very stable; the average for the period between 8/28 and 9/26 was 223 m. This stable trend is 56% greater than the beginning nine-day average of 142 m. After the stable period in September, there was a general decrease in average amplitude for about five weeks. There was a steep decrease in the third week of October after a very gentle trend downward in the beginning of October. The gentle trend continued, but in an increasing manner after the steep fall in mid October. The averaged amplitude near the end of the period was 156 m (based on the data from 8/28 to 11/8). This was a decrease of 30% compared to the relatively stable September value that averaged 223 m.

d. Evolution of zonally averaged wind and the relationship between zonally averaged wind evolution and that of average amplitude

At first analysis of the changes in maximum zonally averaged wind with time in the Northern Hemisphere, there wasn't much change over the period of study. However, there was an overall increase in maximum 500 mb winds from an overall increase in 500 mb winds from 20 September through about 1 November before going back to overall lesser speeds like those of early August. The same pattern also was present in the maximum wind speed in the 300-150 mb layer with an overall increase in speed within the same time period as the max 500 mb wind. This pattern may have occurred because of seasonal change and the arrival of winter in the Northern Hemisphere. A plot of the change in max 500 mb winds and the max 300-150 mb wind over time was made to determine if there was any correlation between the two in the period of study (figure 19). Overall analysis of this graph showed no correlation at all with values of 0.21 for 500mb winds and 0.26 for 300-150 mb layer. At some points the winds increased and decreased with each other and at other times they were inversely proportional to each other. The time series of 500 mb wind with average amplitude (figure 20) didn't show much correlation, with an R^2 value of 0.07, although a recognizable pattern emerged during certain periods like 10-21 October when the 500 mb winds strengthened while the average amplitude decreased. Also, the overall picture of the time series does not show the relationship of the amplitude and 500 mb winds since, for at least half of the period observed, the max 300-150 mb winds and 500 mb winds increased and decreased together. It is expected that as the wave amplitude decreases, 500 mb winds should become stronger since there is less hindrance on the winds, thus allowing them to be stronger, but figure 20 shows that this happens only about half the time. Figures 21 and 22 show the lack of a linear relationship between maximum winds at 500 mb and in the 300-150 mb layer and the latitude at which they occurred in the Northern Hemisphere. R^2 values were 0.02 and 0.03, respectively. Therefore, the latitude at which the highest winds were found at these heights had little to do with the actual magnitude of the winds.

The Southern Hemisphere closely resembled the Northern Hemisphere in terms of the trends discussed for the Northern Hemisphere. The plot of the change of 300-150 mb and 500 mb winds over time (figure 23) didn't show much correlation with R^2 values of 0.09 for 300-150 mb layer and 0.07 for 500 mb winds, but there were two periods where there were patterns in the 500 mb winds. These patterns took place between 2 September and 20 September and between 28 September and about 10 October. During both periods, the overall wind speed decreased from the usual. Then at the end of the period of observation there was a bigger decrease in 500 mb winds. The 300-150 mb winds showed patterns similar to those of the 500 mb winds at about the same time or very close to the same time and also showed a large decrease in speed at the end of the period of observation, too (figure 23). Possible explanations for this include a significant cyclone occurring and a period of strong systems moving through that would decrease the 300-150 mb and 500 mb winds. To see if this was the case, a plot of changes in speed and average amplitudes over time was made (figure 24). This plot showed a little more correlation with lines of regression both decreasing over time for the Southern Hemisphere than that for the Northern Hemisphere. The relationship seemed to be that the speed and amplitudes were inversely proportional to each other since the speed increased when the amplitude decreased and vice versa, which matches what was noted with the Northern Hemisphere, though it is unknown why the correlation there was stronger than in the Northern Hemisphere. Like the Northern Hemisphere, there was a very weak linear relationship between maximum wind at 500 mb and in the 300-150 mb layer and the latitude at which it occurred. Figures 25 and 26 show this, with correlation coefficients of 0.03 and 0.0015, respectively.

4. Conclusions

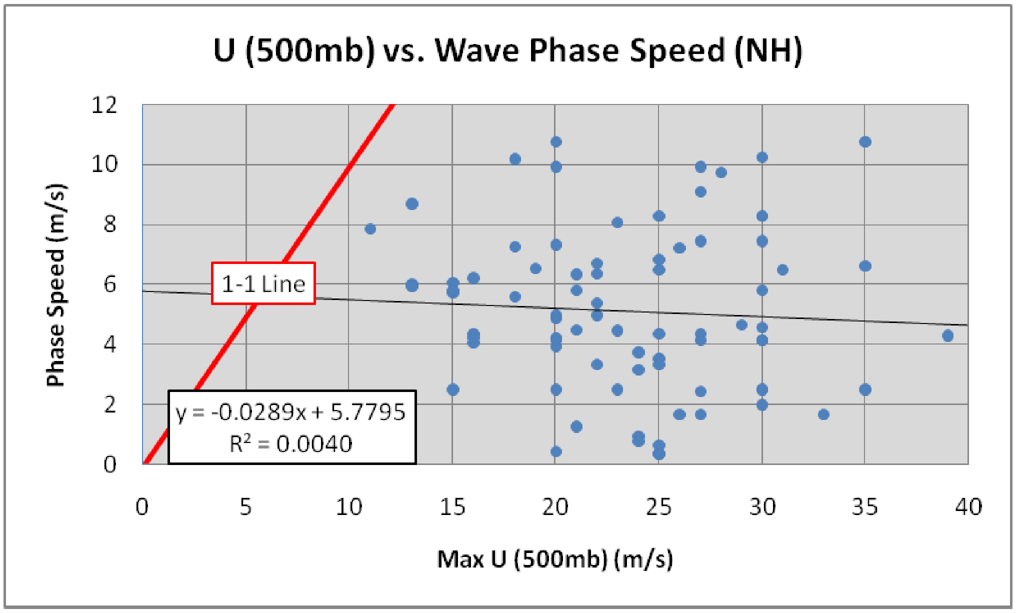
The goal of this study was essentially to test Rossby wave theory: Did the wave patterns observed at the 500 mb level match that suggested by Rossby wave theory? Comparison was made by using the equation for wave speed, $c = \bar{u} - \beta/(k^2 + l^2)$. 500 mb data was gathered in the form of dominant integral wave number, average wave amplitude, wave phase speed, and maximum zonally averaged wind for both the Northern and Southern Hemisphere for a period of 82 days from late August through early November 2008. Analysis was performed using scatter plots and lines (or polynomials) of regression calculated using Microsoft Excel 2007.

After analysis, it is obvious that many aspects of Rossby wave theory did not match the observations. There was no relationship between upper air wind speed and wave phase speed in either hemisphere (or at least if there was a relationship, it was non-linear); there was only a very weak linear relationship between dominant integral wave number and wave speed; there wasn't even a very strong linear relationship between maximum 500 mb winds and maximum winds in the 300-150 mb layer.

Integral wave number did vary with time in noticeable patterns in both hemispheres, but again, there was a very weak linear relationship between average wave amplitude and wave number.

With the help of 3-day and 5-day running averages, time series of average wave amplitude showed interesting wavelike patterns in both hemispheres. There was also a mysterious damping of the wave pattern around the time of the Autumnal Equinox in each hemisphere. Additional study will be required to determine the explanation behind such a phenomenon. Additional time series showed that 500 mb and 300-150 mb winds tended to increase together over the period of study in the Northern Hemisphere, and decrease together in the Southern Hemisphere, albeit the linear relationships were weak. The same was true with maximum 500 mb winds and average wave amplitude in each hemisphere. Plots of maximum winds at 500 mb and in the 300-150 mb layer against the latitude at which they occurred contained absolutely no correlation. These plots were truly "scatter" plots, with correlation coefficients well below 0.10.

Sources of error in the methodology and analysis include: the large subjective factor when discerning the height fields of waves as well as the movement of the waves. It should also be noted that this study evaluated waves at 50 degrees latitude, and those defined by one specific 500 mb height contour in each hemisphere. Although, a mid-latitude location and an average contour for each hemisphere, this is not the only latitude and height at which Rossby wave theory should apply. Isolating this latitude and those contours as a focal point in this study certainly could introduce biases that would affect the apparent agreement between observations and theory. It's also possible that there are no relationships between some of the variables analyzed in the study, or that the relationships are very complicated. The authors suggest that other methods for verifying Rossby wave theory be implemented, such as using other latitudes and contours, using combinations of them, and using other forms of data, such as Fourier analyses, to determine things such as dominant integral wave number.



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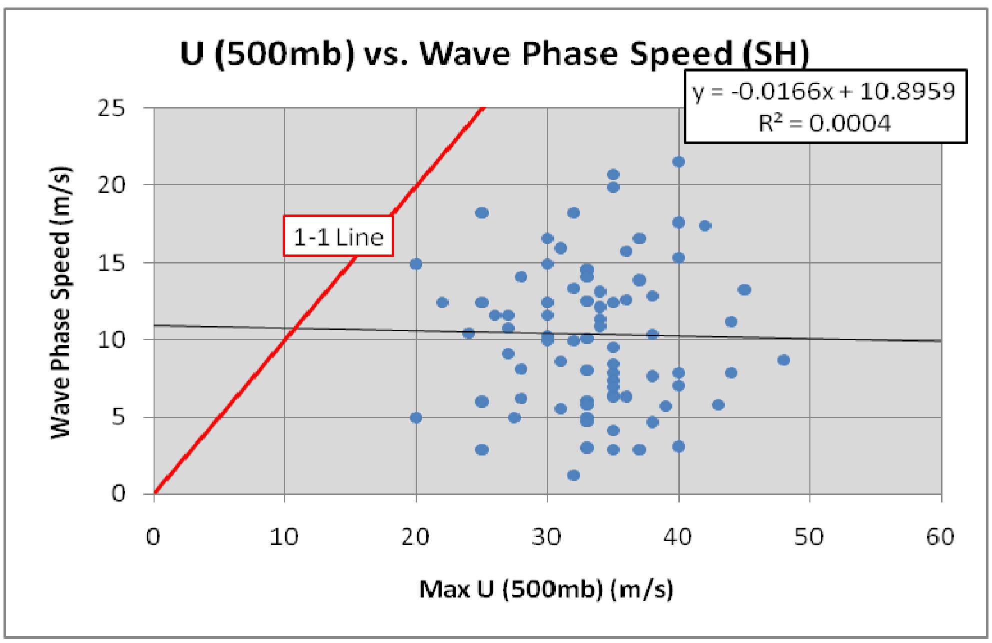

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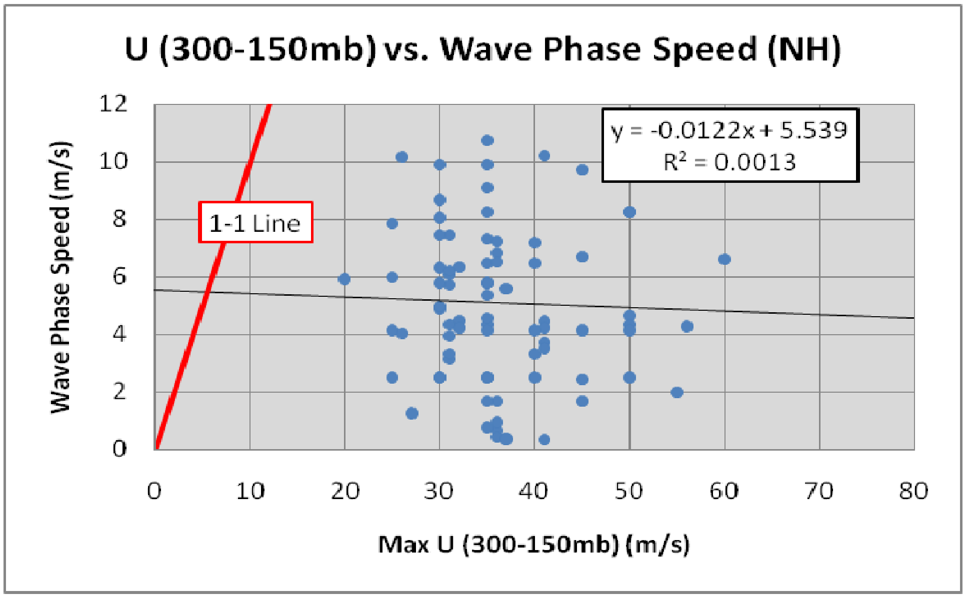

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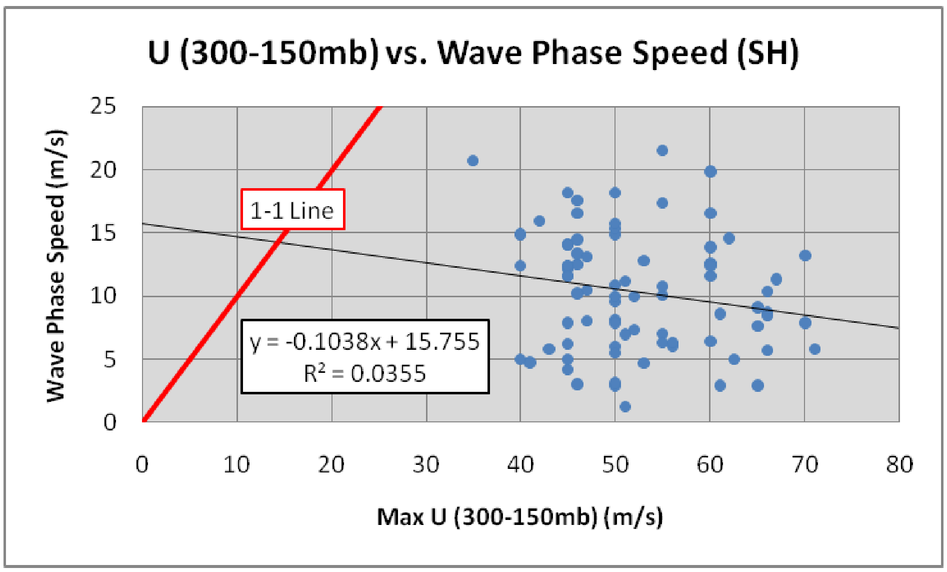

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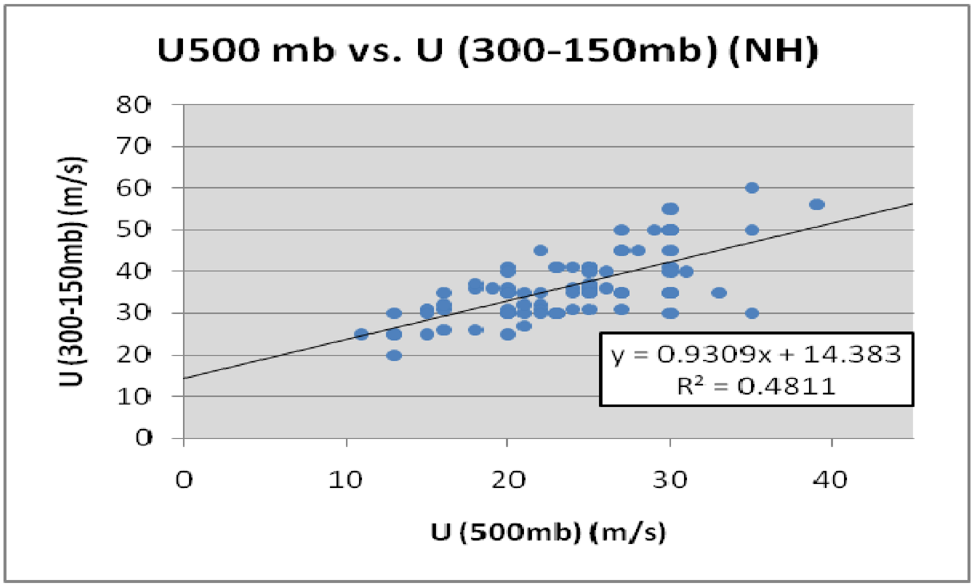

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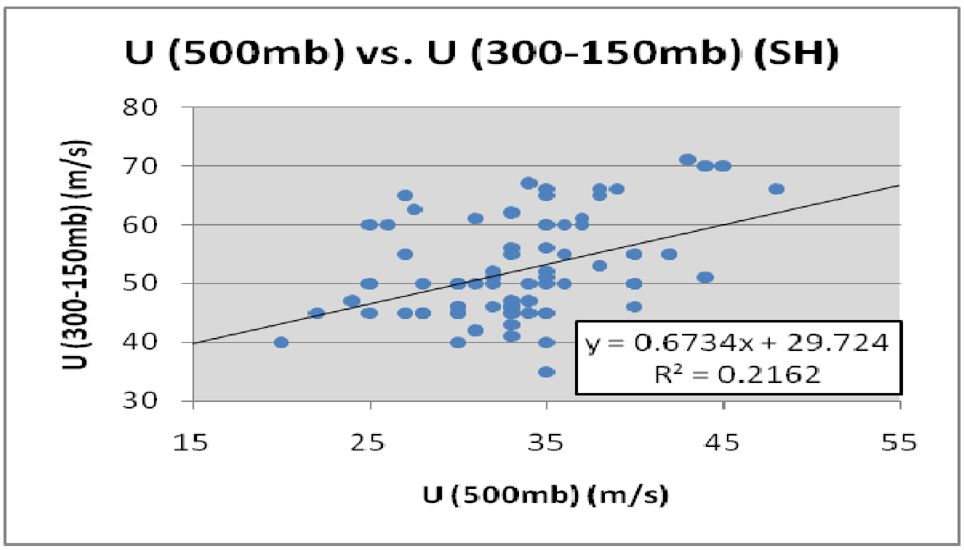

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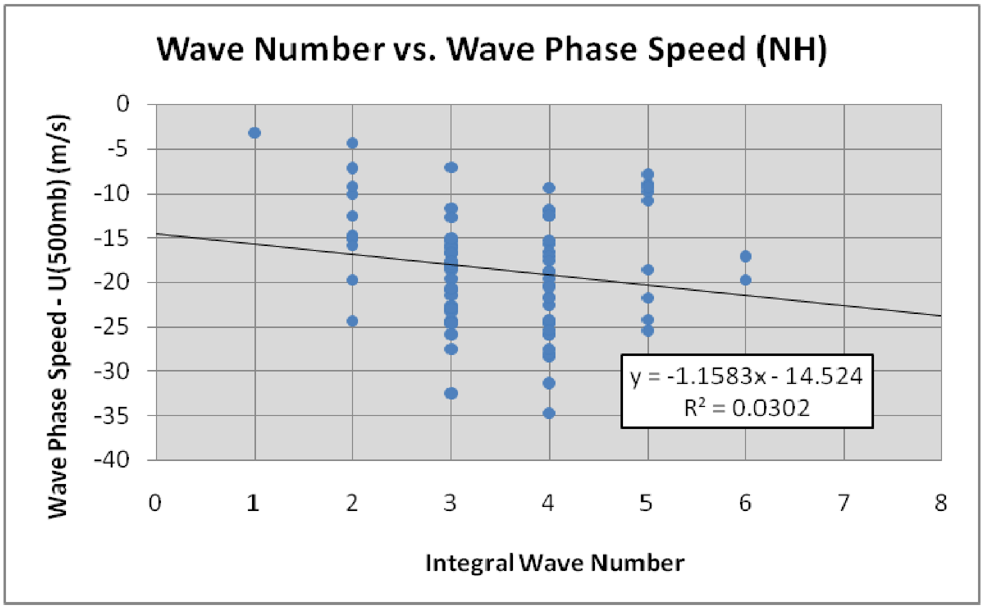

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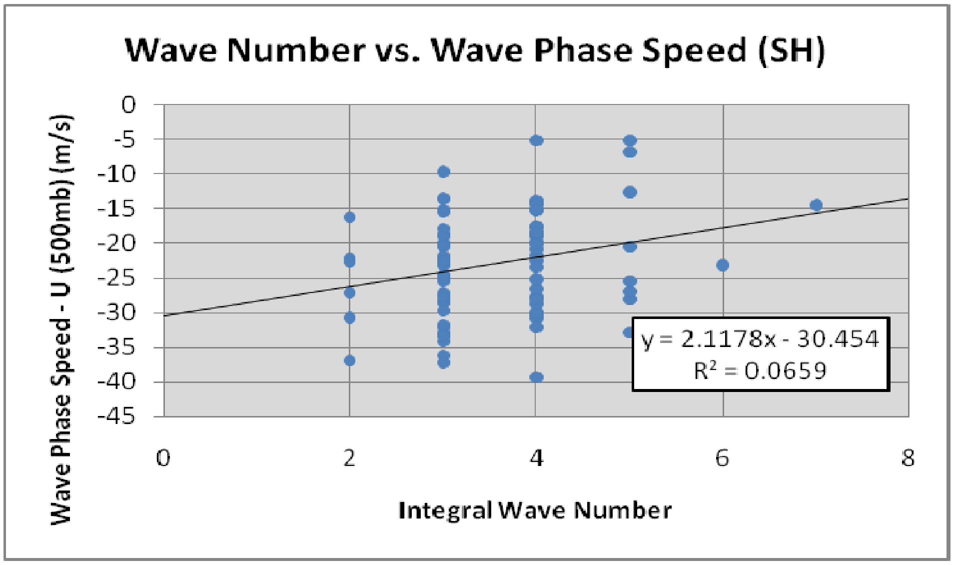

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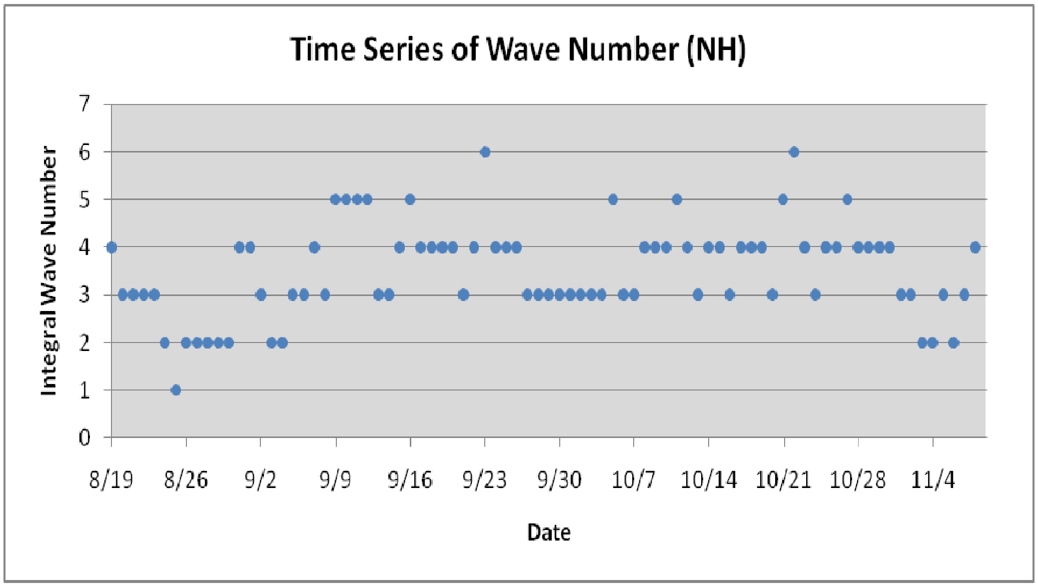

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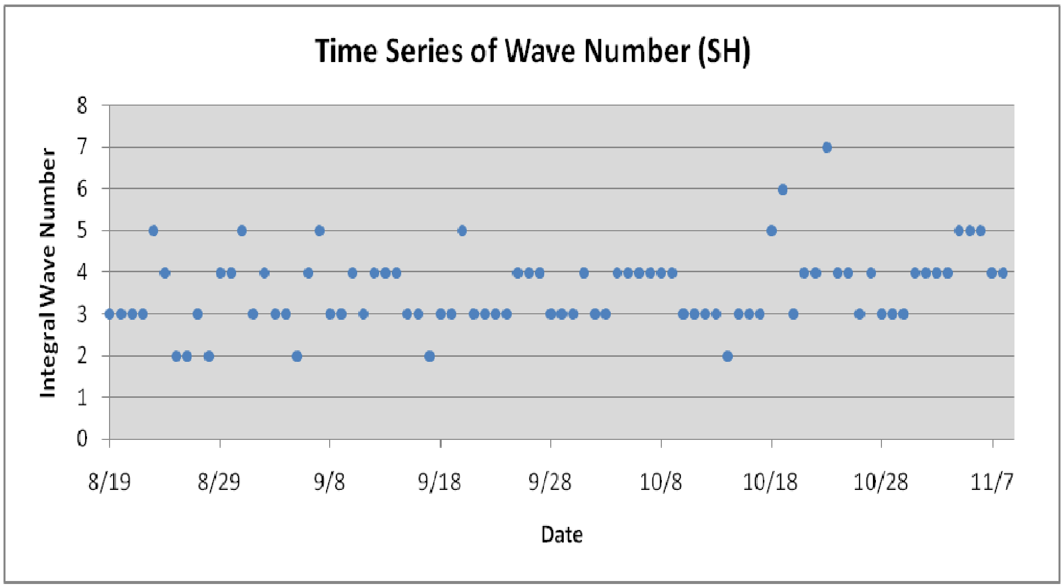

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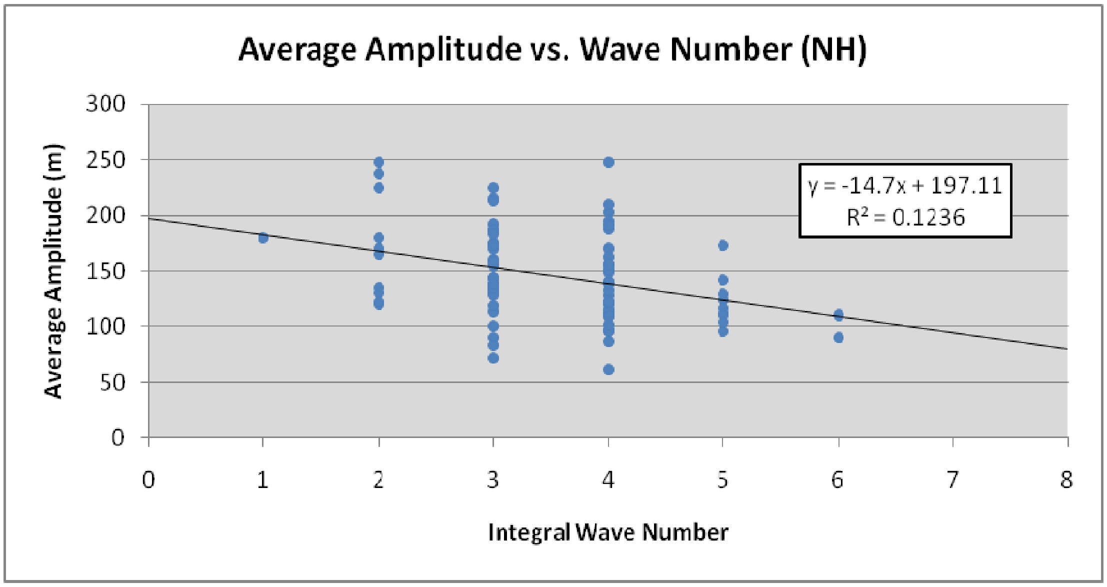

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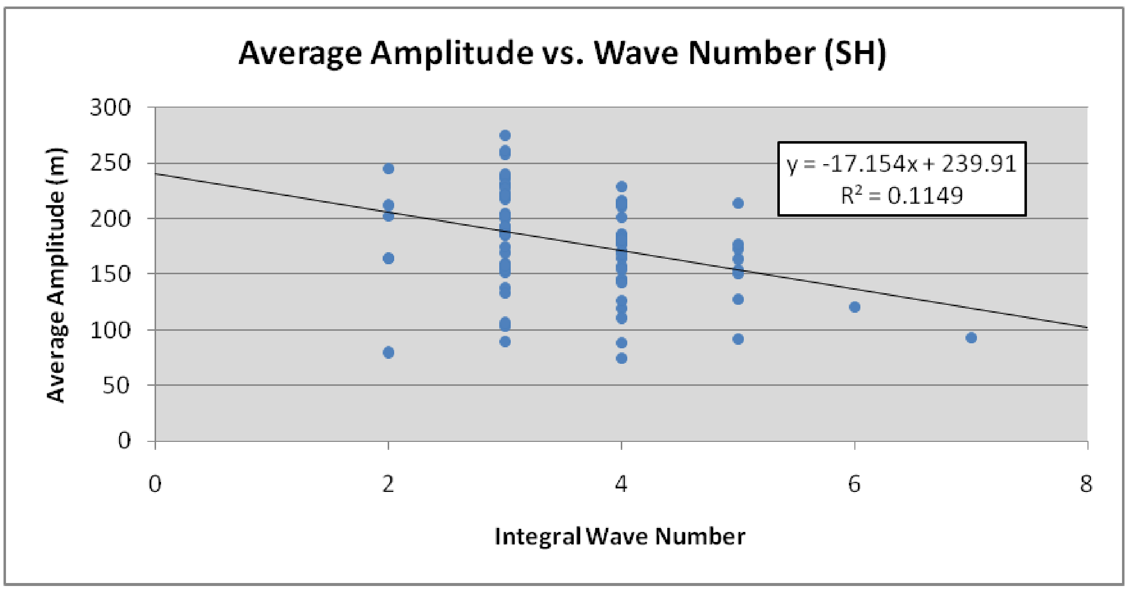

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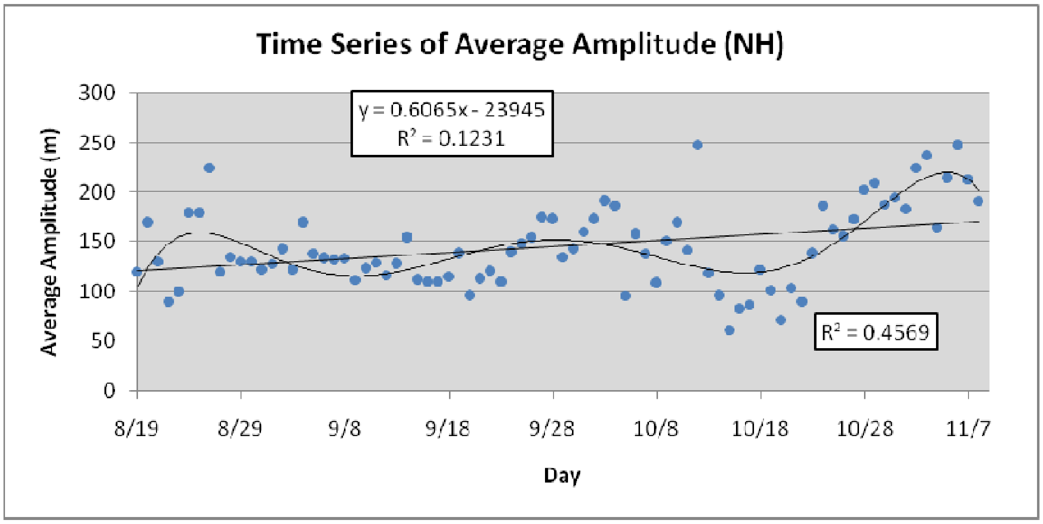

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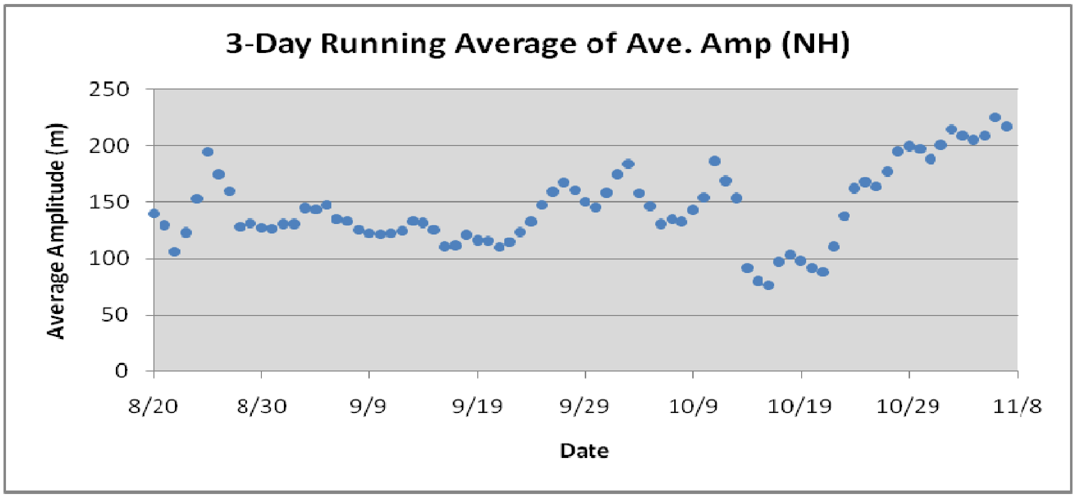

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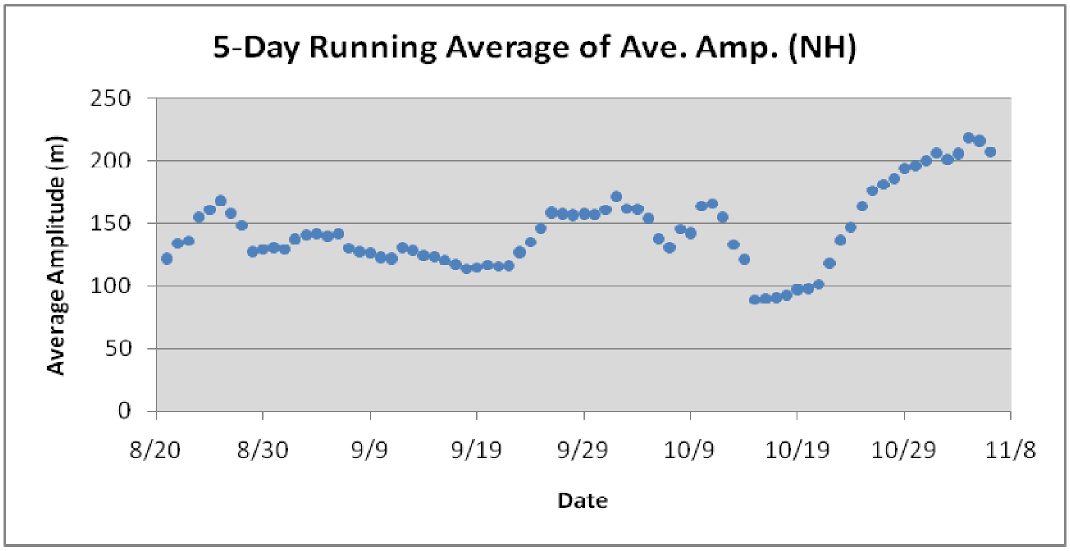

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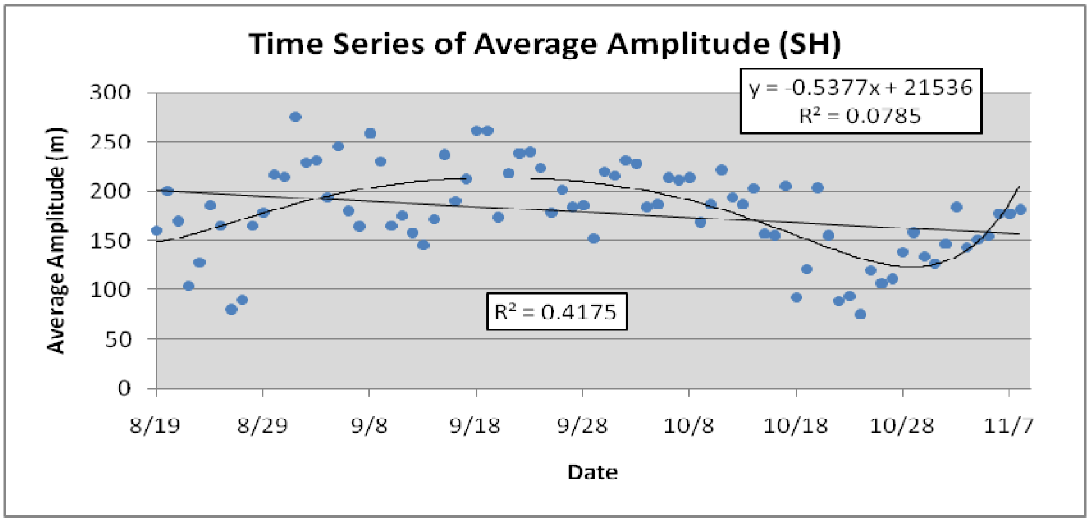

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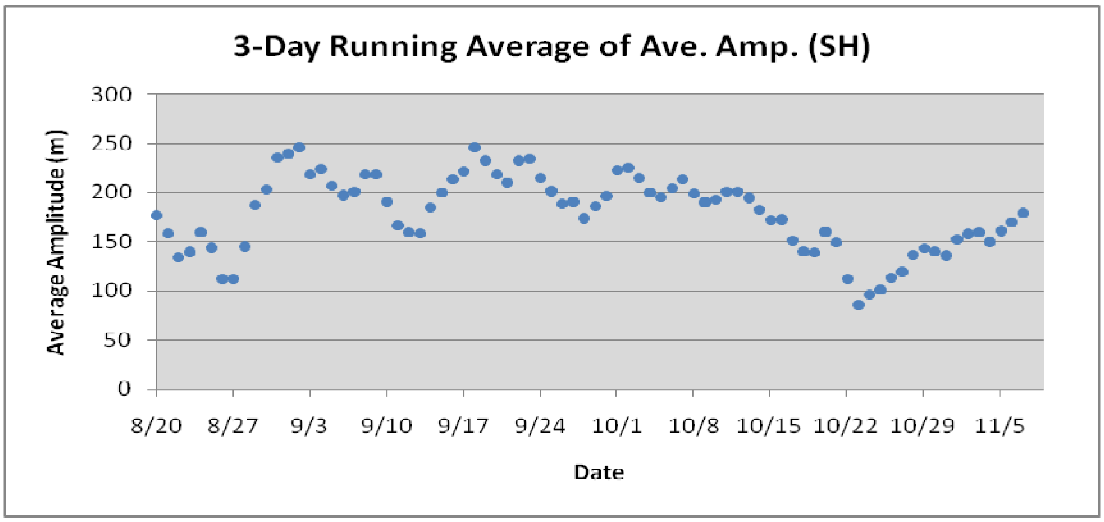

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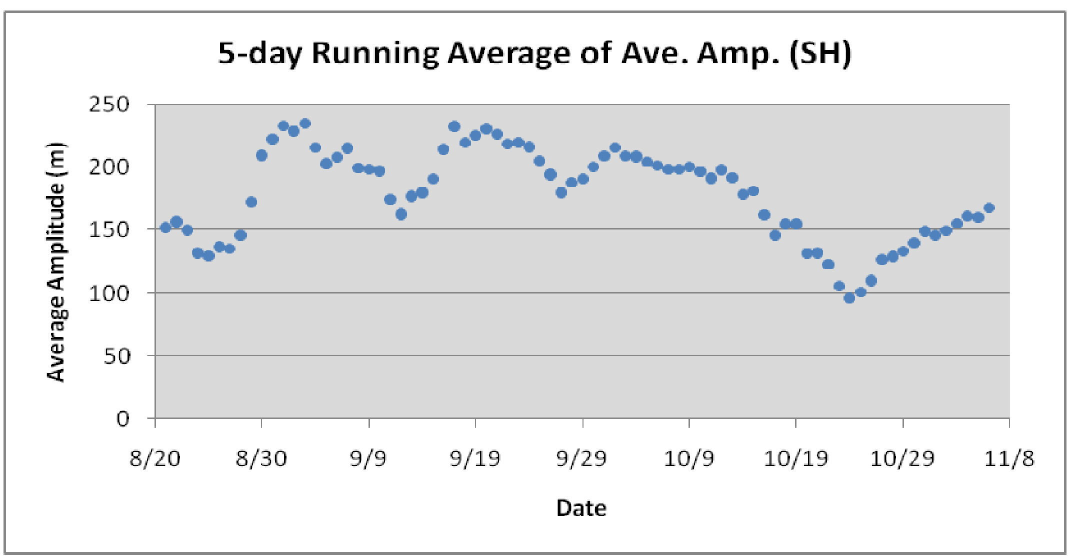

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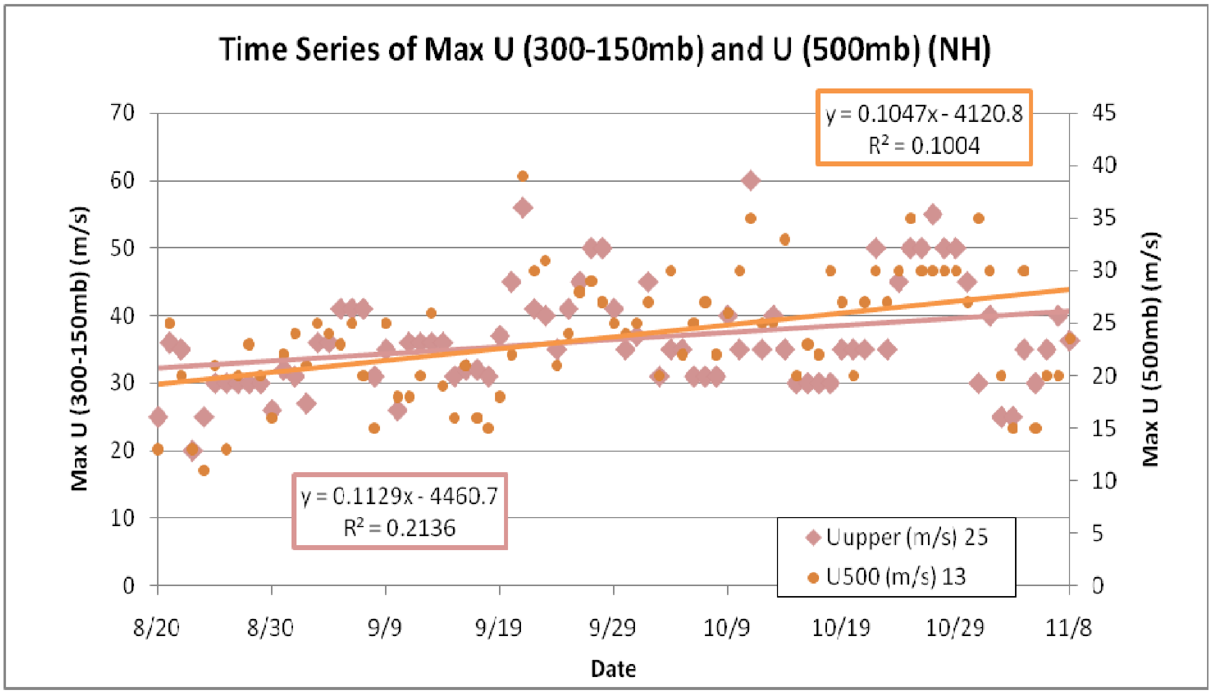

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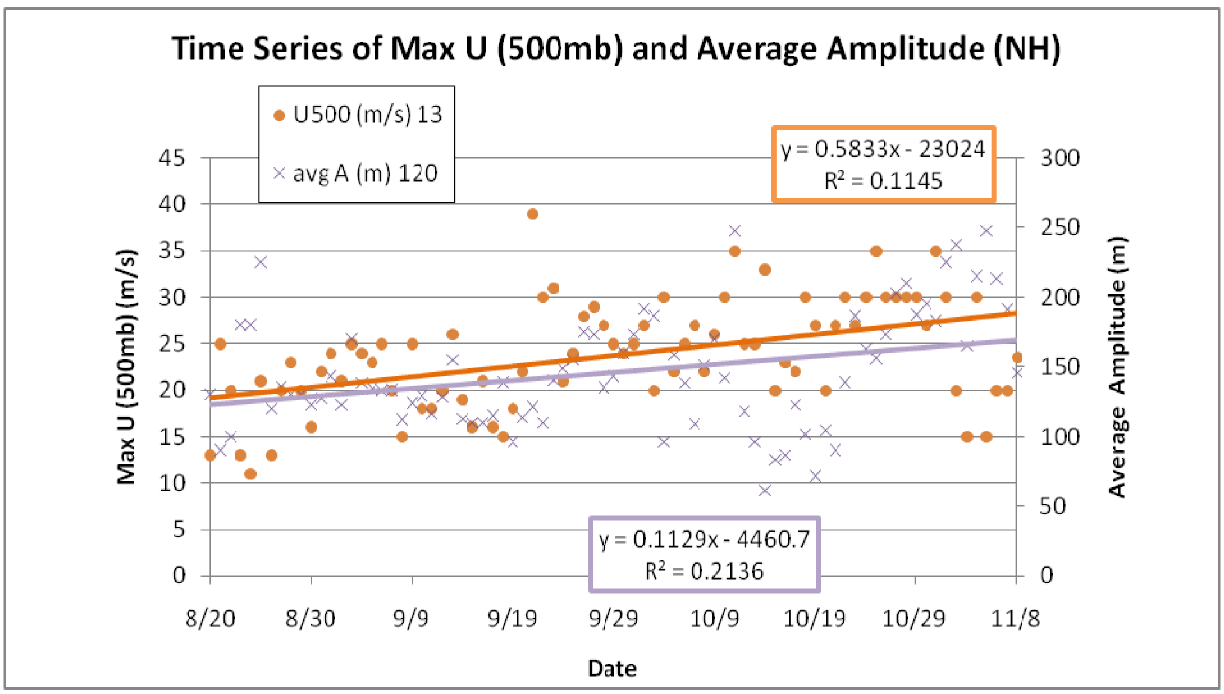

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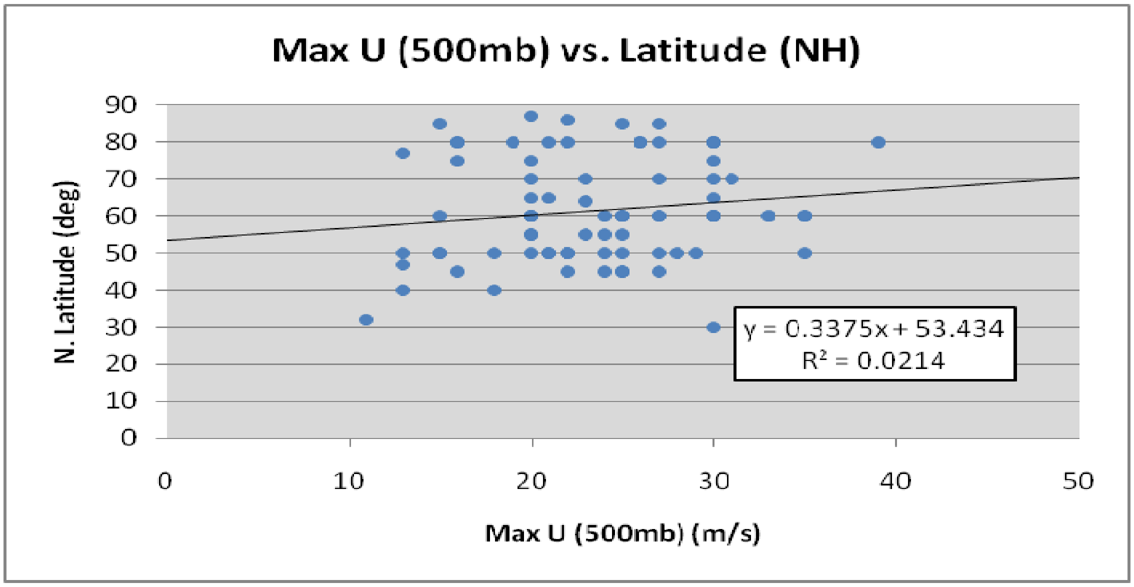

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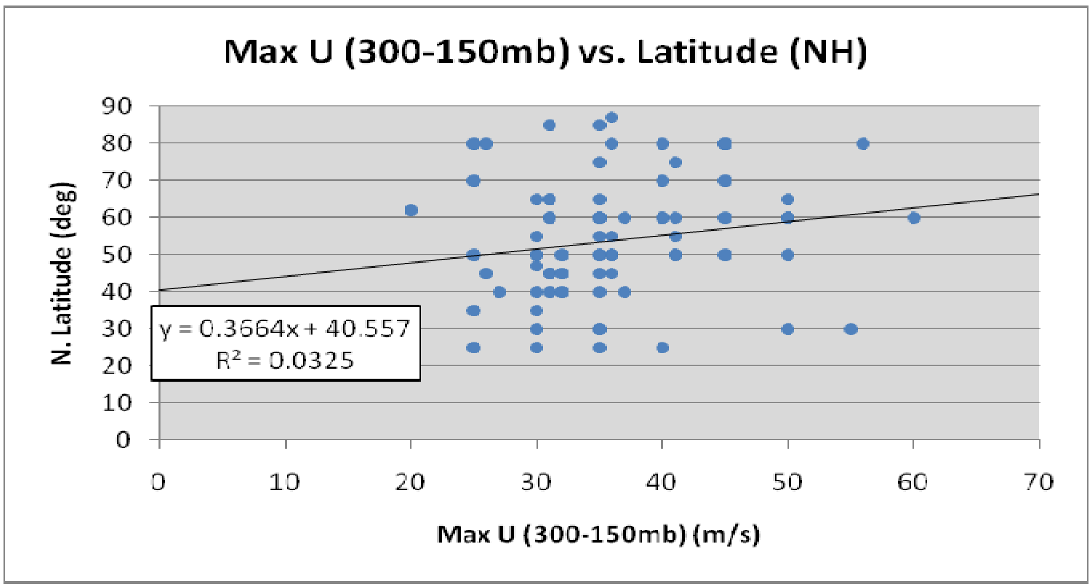

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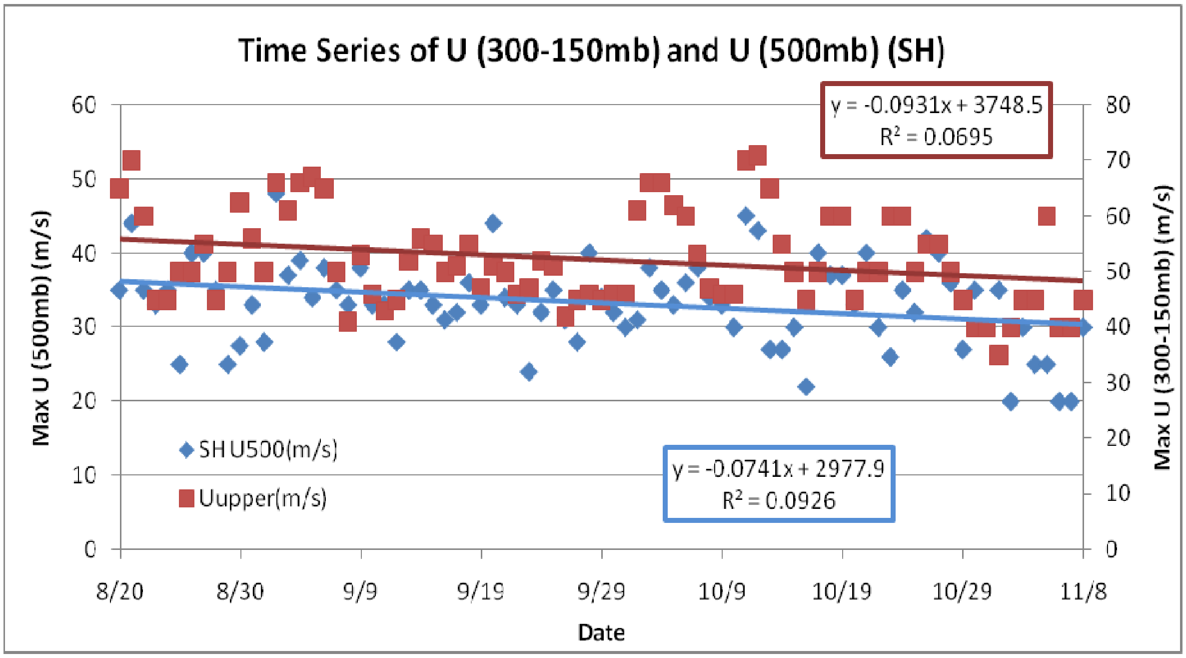

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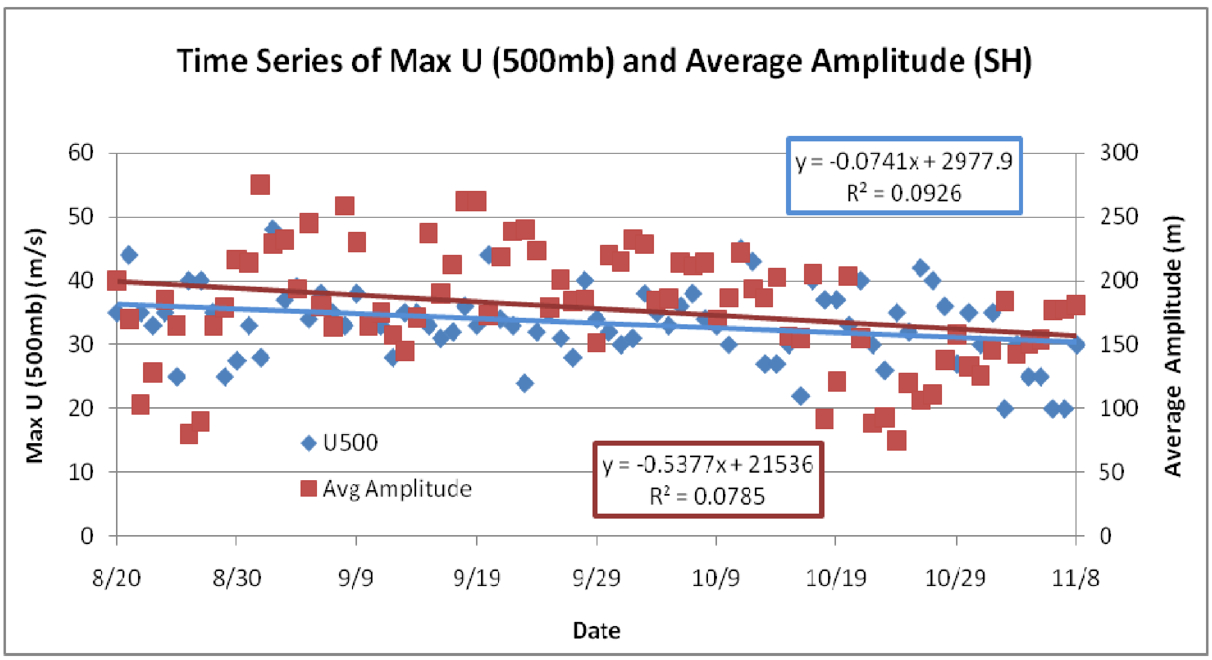

Figure 24

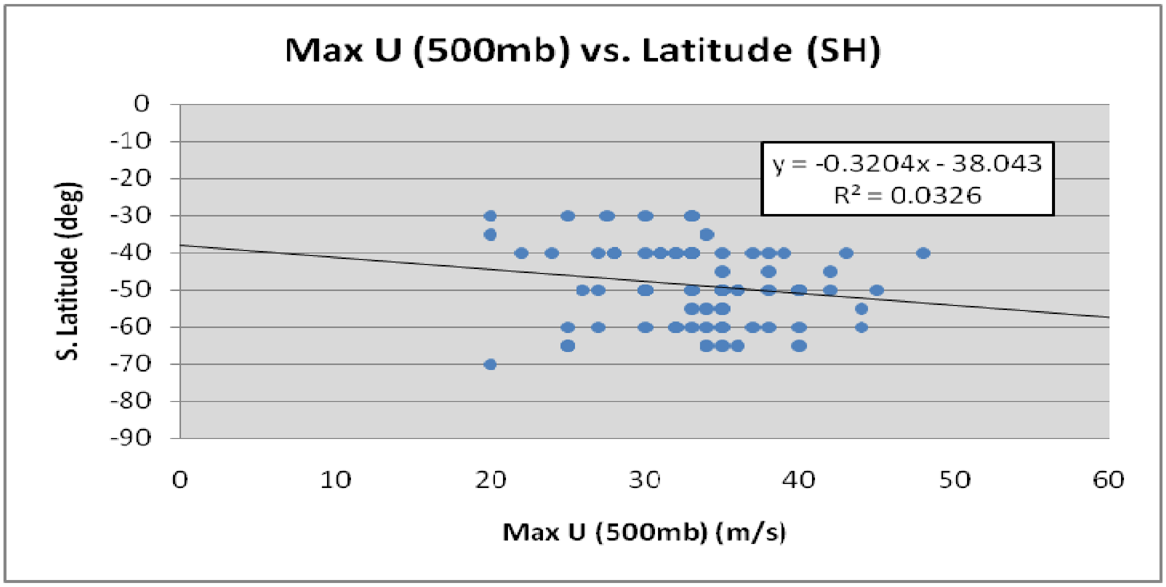

Figure 25

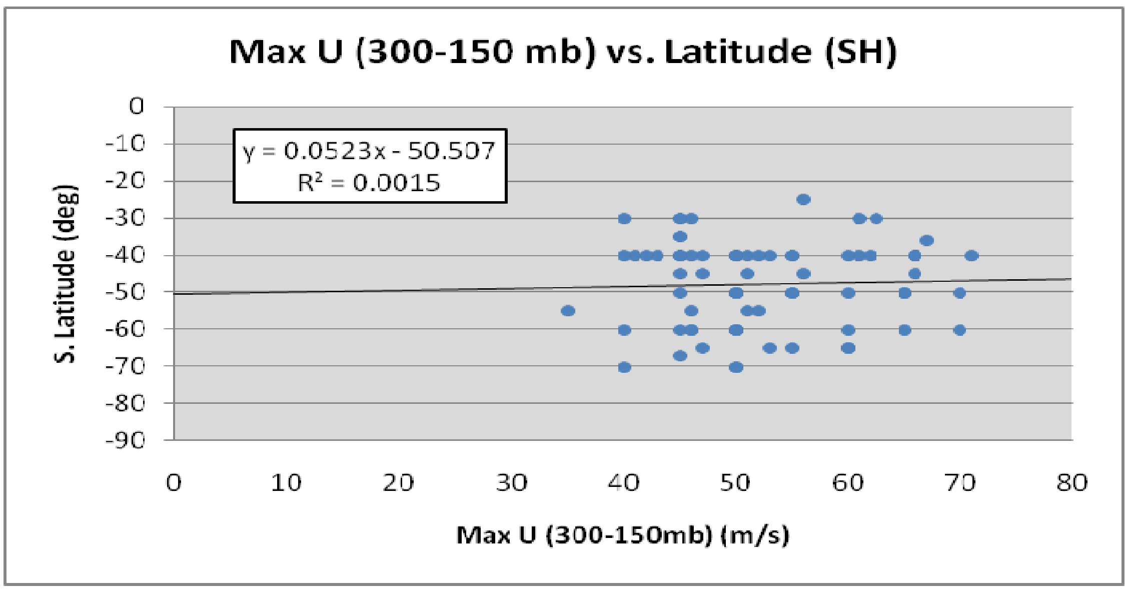

Figure 26