Atmospheric Waves

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ABSTRACT

Studying atmospheric waves helps meteorologist to better forecast the weather. In this study atmospheric waves are studied by analyzing the wave number, motion, amplitude and wind speed which was analyzed at 500 hectopascal and the 300hPa to 150hPa layer. The goal was to prove the Rossby Wave Theory correct by plotting the variables against one another and analyzing the correlation. The results varied when it came to supporting the Rossby Wave Theory, this could be due abnormal wave behavior and limitations both in gathering data and in the Rossby Wave Theory itself.

1. Introduction

Atmospheric waves are important for forecasting because their motion affects the synoptic-scale weather system. Waves with a wavelength greater than 1000 km are considered long, and waves with a wavelength in the range of 100-1000 km are considered short. Both long and short waves cause fluctuations in the synoptic flow in the form of troughs and ridges. Although the average flow is zonal, daily variations in wave trough/ridge patterns arise to affect atmospheric conditions. For example, a trough generates positive relative vorticity and produces strong upward motion and the development of surface low pressure centers. The resulting surface low will generate precipitation and changes in temperature. In this example, it is evident that forcing aloft from a trough in an atmospheric wave can affect the surface conditions. Likewise, a ridge will affect the weather but in the opposite manner. Negative vorticity advection in ridges will yield downward motion and create clearing conditions behind the trough at the surface. Compared to long waves, the trough and ridge fluctuations of a short wave are also smaller in amplitude. This means the dynamic forcing associated with short waves is weaker causing, for example, precipitation or temperature changes at the surface to be smaller and confined to a smaller domain.

Wavenumber, wave motion, and amplitude are three key features of the large-scale atmospheric flow. The wavenumber is the amount of trough/ridge patterns in the flow. Wave motion is best described by the Rossby Wave Theory. The magnitude of wave motion is the phase speed, which is described mathematically:

\[ C_x = v/k = u - \left( \frac{\beta}{k^2 + \ell} \right) \]
in which \( C_x \) is the phase speed in the x-direction, \( u \) is the zonal wind, \( \beta \) the variation of planetary vorticity with change in latitude, \( k \) the wavenumber in the x-direction and \( l \) the wavenumber in the y-direction. Several important relationships can be seen from this equation. As the positive zonal wind speed increases, the phase speed will increase in the positive x-direction. If the zonal wind is small enough to be considered negligible, the phase speed will be in the negative x-direction. This negative x-direction motion is the true nature of long-wave flow in the absence of a zonal wind. Usually the zonal wind is strong enough to force the wave to propagate in the positive x-direction, but in some cases a retrogression of the wave will occur when the positive zonal wind speed is small. For example, the zonal wind speed above a cut-off low pressure system is generally small because it is disconnected from the main jet stream flow aloft. In general, systems that are not cut-off propagate to the east (positive x-direction) because the zonal flow is usually strong enough to force the system from its westward (negative x-direction) motion. It is also important to take note that as wavenumber decreases phase speed increases in the presence of a zonal wind from the west. This is why long waves have slower phase speed compared to short waves because of their larger wavenumber. Finally, amplitude shows the amount of energy present in a wave. A wave with a larger amplitude will contain more energy, which is the reason long waves are stronger, produce more dynamic forcing than short waves.

Besides daily fluctuations, seasonal variations in atmospheric waves exist. There are two annual modes: spring/fall and summer/winter. During the spring and fall, the wavenumber is greater than in the summer and winter. The spring/fall mode also has a larger change in intensity compared to the summer/winter mode. A weakening and strengthening of waves is observed in the spring and fall, respectively.

Three hypotheses were made at the beginning of the project: 1) Stronger zonal winds at the 500 mb and 300-150 mb levels would produce faster wave motions, 2) faster wave motion will result from a larger wavenumber, and 3) a larger wavenumber yields a lower amplitude. The project focused on collecting wavenumber, phase speed, and amplitude data daily to test each hypothesis.

2. **Data and Methods**

   a. **Waves**

Wave data was acquired from the Iowa State Weather Products website. Data was analyzed from September 1, 2013 through November 15, 2013. The products used on this site were the Northern Hemisphere 500 hectopascal (hPa) and Southern Hemisphere 500 hPa analysis and the global zonal average wind. Analysis for both the Northern and Southern latitude were centered along the 50° latitude line. The data obtained was wavenumber, amplitude, and wave motion. To measure these values the 5580 meter (m) contour was used for the Northern Hemisphere and the 5280 m contour for the Southern Hemisphere. The wavenumber, \( N \), was calculated by averaging the number of times the 5280/5520 m contour crossed the 50° latitude
Amplitude, A, was calculated in this study by equation # where $A_{ridge}$ is the average trough amplitude and $A_{trough}$ is the average ridge amplitude.

\[
(2) A = \frac{(A_{ridge} + A_{trough})}{2}
\]

\[
(3) A_{ridge} = \frac{(A_{r1}+A_{r2}+...+A_{rn})}{N}
\]

\[
(4) A_{trough} = \frac{(A_{t1}+A_{t2}+...+A_{tn})}{N}
\]

Where $A_{r1}, A_{r2},$ and $A_{t1}, A_{t2}$ are the individual wave ridge and trough amplitudes. The distance the 5580/5280 m contour for Northern and Southern Hemisphere respectively extended below the 50° latitude line was considered the amplitude for the trough or ridge.

Wave motion, C, was determined by measuring the distance a wave traveled over a three day period.

\[
(5) C = \frac{\{LON(day+1)- LON(day-1)\}}{2}
\]

This was done by measuring the starting longitude of the day before and the longitude of the following day and averaging the two values. The wave motion was measured in degrees longitude.

b. Zonal Wind

Zonal wind data was collected by determining the zonal wind at 500 hPa and the maximum zonal wind at the 150 hPa-300 hPa layer. This data was gathered at the 50° latitude line for both the Northern and Southern Hemisphere. Positive zonal wind directions mean an eastward wind, thus negative values are a westward wind.

c. Limitations

With all of these calculations there are limitations. For determining the wave number if the 5580/5280 m contour does not reach or cross the 50° latitude line it was excluded. Another limitation was in calculating amplitude. The amplitudes for the ridges and troughs are estimations, the contour lines were hard to read at times especially in a deep ridge or trough. When it came to calculating the wave motion waves that weakened significantly or barely moved were ignored. The final calculation with errors is in the zonal wind, since the 50° latitude line was not clearly marked it was estimated roughly where it would be and calculations were taken from there. Also sometimes a strong gradient made it difficult to accurately read the wind values because lines were very cluttered.
3. Results  

a. Zonal Wind

The zonal wind plays a major role in wave motion. A zonal wind from the west will cause a wave to propagate to the east, while a zonal wind out of the east will cause westward propagation. The Northern and Southern Hemispheres have different zonal winds. This is evident in Figure 1 in which the 500 hPa zonal wind is plotted with the date.

Overall, the Southern Hemisphere zonal wind has a larger magnitude compared to the Northern Hemisphere. Similarly, the average zonal wind in the Southern Hemisphere is greater than in the Northern Hemisphere. A plausible explanation for the smaller zonal wind magnitude and average in the Northern Hemisphere is frictional dissipation. The 50° N line in the Northern Hemisphere crosses a greater landmass than at 50° N in the Southern Hemisphere. With more topography comes greater friction that forces the speed of the upper level winds to decrease. Also, the greater range of zonal wind magnitudes in the Southern Hemisphere can be explained in terms of friction. The larger frictional drag in the Northern Hemisphere causes faster winds to decrease more than slower winds. This effect results in a smaller range as there is less variance
in the wind speeds. With most of the 50° N line over the ocean in the Southern Hemisphere, there is less friction and the upper level zonal wind is stronger and varies more. The Northern Hemisphere featured an increasing, positive zonal wind trend over the entire data period. Contrastingly, the Southern Hemisphere had a decreasing, negative trend. These features in the data disagree with the fall weakening and spring strengthening of the zonal wind that is normally observed. A possible explanation is the annual spring (fall) mode that decreases (increases) the intensity of waves. Human error in calculating the zonal wind magnitudes is another possible explanation for the disagreement. If smaller (larger) values would have been computed for the Northern Hemisphere (Southern Hemisphere) toward the end of the period, then the trend would have been decreasing (increasing). The larger temperature gradient in the spring should have caused the Southern Hemisphere to have an increasing, positive trend. Oppositely, the weaker temperature gradient in the fall should have made the zonal wind speed trend decreasing and negative in the Northern Hemisphere.

In the Northern Hemisphere, a similar trend exists in the magnitude of the zonal wind speed and the wave amplitude. The positive, increasing trend over time can be seen in Figure 2.
Since wave amplitude is a measure of the amount of energy in a wave, the increase in amplitude shows this intensity trend. This agrees with the characteristic of waves to intensify in the fall. However, the relationship between the increasing amplitude and zonal wind speed disagrees with Rossby Wave Theory. A stronger zonal wind should produce a faster wave motion (phase speed), thereby creating a greater wavenumber which in turn yields a smaller amplitude. Despite this disagreement, the daily/weekly fluctuations of the zonal wind and amplitude seem to correspond well. An increase (decrease) in the zonal wind is associated with an increase (decrease) in wave amplitude. The magnitude of these fluctuations is also comparable in magnitude, relatively.

In the Southern Hemisphere, the relationship between the zonal wind speed and wave amplitude is opposing. It contrasts the Northern Hemisphere trend because the zonal wind speed decreases over time. The wave amplitude increases from winter to spring. Figure 3 depicts this relationship.

![Figure 3](500hPa Zonal Wind vs. Amplitude SH)

The opposing relationship agrees with Rossby Theory. As the zonal wind speed decreases, the wave motion (phase speed) becomes smaller, which causes the wavenumber to
become smaller as well. A smaller wavenumber then yields a larger wave amplitude. As in the Northern Hemisphere data, the daily/weekly fluctuations in the zonal wind and wave amplitude correspond. An increase (decrease) in the zonal wind is related to an increase (decrease) in the wave amplitude, relative to scales. These small-scale fluctuations were embedded within the large-scale trends. The gradual increase or decrease in the daily/weekly fluctuations appears to have caused the overall trend for both the zonal wind and amplitude data.

b. Wave Amplitude

The amplitude varies with time throughout the dataset. In the northern hemisphere there almost seems to be a cyclical motion rising for three or four days and then falling for two or three days as shown in Figure 4 with the general rising trend overall (22% increase). This is reasonable as will be described later in the amplitude vs wave number section. There were sharp rises towards the end of our data from October 27 to the 29th the amplitude fell from 330 m to 195 m (26%) and then again from November 12 to the 14th the amplitude fell from 340 m to 217.5 m (22%) and continued to fall as we ended our data set.

![Figure 4](image)

Amplitude vs. Date NH

\[ y = 1.1146x + 152.43 \]

\[ R^2 = 0.2594 \]
The southern hemisphere was not as cyclical, and towards the end of the data set seemed to fit the line much closer than initially observed evident in Figure 5. The southern hemisphere experienced a slight downward trend (~0% fall). Along with this downward trend there were two significant falls in the southern hemisphere towards the beginning of the data set the first is quite significant because it reaches from the maximum amplitude to the lowest amplitude for the southern hemisphere data set from September 1 to the 6th the amplitude fell from 360 m to 125 m (48.5%). The second fall while still impressive was slightly less drastic ranging from 345 m to 140 m (42.3%) with these drastic changes and the general average in between the two is possibly why our line has no significant slope to it.

![Amplitude vs. Date SH](image)

There seems to be an interesting outcome when the two hemispheres amplitudes there seems to be portions of the time when the northern and southern hemisphere are in phases (September 3 through the 8th and November 4 through November 11th as shown in Figure 6) and out of phase the rest of the data set when the data is out of phase it seems to be by a range of 2 - 3 days afterwards as shown in Figure 6.
c. Wave Number

The wave numbers, just as the amplitudes, varied with time throughout the dataset, but in each hemisphere the most common wave number was three. The northern hemisphere dataset started out with lower numbers in general began to grow through the end of September and dropped down again towards the end, where the southern hemisphere had a slight downward trend with the highest values in the middle and the lowest near the end of the dataset as seen in Figure 7 and Figure 8. The overall trends of these graphs can be explained in part due to seasonal changes. As the northern hemisphere moves out of the summer (less active season) and into the winter season the number of the waves generally increases. The opposite is occurring in the southern hemisphere and the data shows that much more clearly. The average wave number for the northern hemisphere was 3.82 while the southern hemisphere had an average of 3.05. In some cases between spikes or peaks there are periods of 7 or 14 days where there is generally a dominant wave number with a +/-1 in Figures 7 & 8 this agrees with the general synoptic timescale of 7 days.
Figure 7
Wave Number NH vs. Date

y = 0.0013x + 3.7924
R² = 0.0005

Figure 8
Wave Number SH vs. Date

y = -0.0078x + 3.3604
R² = 0.0328
The amplitude when graphed against the wave number shows that there seems to be a trend in both hemispheres that the lower the wave number the higher the amplitude, and vise versa Figure 9 & 10. This is a good indication that the more waves you have the less intense the amplitude is for that hemisphere where if you have less waves they will have much more energy thus resulting in a higher amplitude.

![Amplitude vs. Wave Number NH](image)

- Equation: $y = -17.313x + 261.86$
- $R^2 = 0.1918$

Figure 9
d. Wave Motion

Through analysis, we found that there is a correlation between wave number and wave speed. In the northern hemisphere, wave speed tends to increase as wave number increases, averaging a wave speed of about 10 deg/day at a wave number of 1 to about 12 deg/day at a wave number of 6 (Figure 11). This agrees with the hypothesis as well as Rossby Wave Theory, which states that a smaller wave number (k) will result in a larger $\beta$ term, causing an overall slower motion. The opposite is true for larger wave numbers. This can be seen in nature, since short waves travel faster than long waves.

The southern hemisphere proved to be a bit more abnormal. As wave numbers rose from 1 to 5, the average wave speed remained nearly constant at about 14 deg/day. This does not agree with Rossby Wave Theory or the hypothesis, both of which state that wave speed should increase with wave number. There are several reasons for the disagreement between the data and what was expected. One of the biggest contributors could have been sampling errors. Due to the nature of how the data was collected, it is quite possible that waves that were barely below the 50 degree longitude threshold were ignored, thus resulting in a misrepresentation of what actually happened. Additionally, Rossby Wave Theory assumes that waves occur in barotropic flow (the waves have the same density throughout). This may not be the case in the real atmosphere.
Wave Motion vs. Wave Number NH

\[ y = 0.3451x + 9.8979 \]
\[ R^2 = 0.0188 \]

Wave Motion vs. Wave Number SH

\[ y = -0.0477x + 13.955 \]
\[ R^2 = 0.0001 \]
From the data that was collected, it appears that there is a relatively minor positive correlation between wave motion and 500 hPa zonal winds. Both the Northern (Figure 13) and Southern (Figure 14) Hemispheres show this relationship. Even though the correlations for the linear fits are not strong (0.0006 for the Northern Hemisphere and 0.0171 for the Southern Hemisphere), the overall trend is for wave motion to be more eastward as eastward zonal winds increase. Likewise, motion is seen to become more westward as zonal wind becomes westward. This agrees with Rossby Wave Theory, which states that with no zonal wind at 500 hPa, wave motion would be to the west. Again, while the overall trend appears to agree with Rossby Wave Theory, the linear fit for the data is not very strong. A cause for this could have been the way wave motion was recorded and calculated, leading to errors.

Figure 13
Additionally, the relationship between wave speed and 500 hPa zonal wind speed agrees with Rossby Wave Theory. The theory states that as eastward zonal wind increases, eastward wave motion also increases. The average 500 hPa zonal wind in the Northern Hemisphere was about 10 deg/day, which is much less than the average 500 hPa zonal wind in the Southern Hemisphere, about 20 deg/day. The positive relation is reflected in more eastward wave motion (about 14 deg/day) in the Southern Hemisphere than in the Northern Hemisphere (about 12 deg/day). However, the motion of the Northern Hemisphere waves do not agree with Rossby Wave Theory in one aspect. According to the theory, waves should propagate eastward slower than the eastward zonal flow. This is not the case for the data that was collected.

Wave motions are generally eastward in the Northern and Southern Hemispheres, however the motion in the Northern Hemisphere is smaller than the motion in the Southern Hemisphere. With averages of 12 deg/day (NH) and 14 deg/day (SH), it would take a wave about 30 days to travel around the earth in the Northern Hemisphere and about 25.7 days in the Southern Hemisphere. This means that weather systems more than likely travel faster in the Southern Hemisphere, possibly affecting areas for a shorter period of time than in the Northern Hemisphere.

Finally, the wave motion has a positive (albeit weak) relationship to the upper-level (150-300 hPa) zonal winds. Overall, as the upper-level zonal winds increased, the eastward wave motion increased. This agrees with Rossby Wave Theory, just like the 500 hPa zonal flow. This is true in both the Northern (Figure 15) and Southern (Figure 16) Hemispheres.
Wave Motion vs. 150-300 hPa Max Wind NH

\[ y = 0.0033x + 11.155 \]
\[ R^2 = 0.0001 \]

Figure 15

Wave Motion vs. 150-300 hPa Max Wind SH

\[ y = 0.0431x + 12.303 \]
\[ R^2 = 0.0104 \]

Figure 16
e. Thermal Wind

The thermal wind relationship states that changes in geostrophic height will be present near horizontal temperature gradients, with higher winds being located higher in the atmosphere. The latter relationship is due to decreasing friction with height. The collected data strongly confirms this relationship, with greater wind speeds almost always occurring in the upper-levels (150-300 hPa). This is true for both the Northern (Figure 17) and Southern (Figure 18) hemispheres.

Figure 17
4. Conclusions

Throughout our study the results vary with the expected results described by the Rossby Theory. Disagreement with Rossby’s theory appeared with zonal wind vs. amplitude in northern hemisphere, zonal wind vs. date, wave motion vs. wave number in southern hemisphere. Agreement occurred with wave motion vs. wave number northern hemisphere, wave number vs. amplitude. Additionally, one out of three of our hypothesizes was completely supported by our findings. A larger wavenumber was shown to yield a lower amplitude. That faster wave motion results in a larger wavenumber was not fully proven and neither was stronger zonal winds produce faster wave motions. The main reasons behind these disagreements would have been errors in gathering data and wave variations for the time period. It can be concluded that the Rossby Wave Theory is not perfect but it does help meteorologists to forecast the weather a little more accurately.

References